Feasibility study for high bit rate pulse position modulation in fibre communications

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Abstract

With the rapid progress of optical communication systems, capacity limitation becomes important in high bit rate links. Overcoming this limitation may involve using different modulation and multiplexing methods, including RZ modulation with on-off keying or pulse position modulation. In soliton links, the capacity limitation at high bit rates is likely to be caused, not by the widely studied Gordon Haus Jitter, but by soliton interaction. In this project, a semi-analytical model for the soliton interaction effect on the maximum length of a solitonic link is developed. Soliton interaction is shown to decrease the transmission distance substantially for links using either on-off keying or pulse position modulation; on-off keying maintains superior performance under all conditions studied. The prospects of pulse-position modulation for linear (non-solitonic) RZ links are also assessed briefly, and some methods of practical implementation of PPM at high bit rates suggested.

Keywords: soliton communications, soliton interaction, pulse position modulation, system length, bit error rate.

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Author's Declaration

I hereby declare that this thesis is a result of my own work and all contributions from external sources have been explicitly stated and referenced.

Parts of the research have been presented in a conference paper (*Feiyue Ji and E.A.Avrutin, "Gigabit-rate pulse position modulation: applicability and possible realisation by methods of integrated optoelectronics", in: Programme and Abstracts, International Conference on Semiconductor and Integrated Optoelectronics, Cardiff, 9-11 April 2013, p. 35*), and submitted for publication in *Microwave and Optical Technology Letters*.

The software used in the simulations was either used under University licence or was shareware.

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Chapter 1 Introduction

My project topic is Feasibility study for high bit rate pulse position modulation in fibre communications. This project is focused on the maximum transmission distance of the two schemes based on generally the same initial power and other affections.

In this chapter, the background information on on-off keying and pulse position modulation applied in optical communications will be introduced and explained, including the principle and classification. Also the structure of this report will be stated in the last section.

1.1 Optical Communications Technology

The technology of optical communications transmits information by using light to carry it over a certain distance. Information is encoded in the form of optical pulses. This technology is most commonly using optical fibre as the transmission medium; though planar monolithically integrated optical waveguides are used within computers and communication equipment which normally based on a very short distance in the laboratory, and free-space optical links are also possible (see below). Light-emitting diodes (LEDs) or laser diodes are commonly used as transmitters in optical fibre links. Due to less attenuation and dispersion experienced when optical fibres transmit infrared wavelengths, infrared light, most typically at a wavelength around 1300 or 1550 nm, is used more usually rather than visible light. Although both optical phase and frequency modulation have been established in the laboratory experiments, in practice intensity modulation is naturally the simplest way of encoding signal. With the requirement of significantly lower cost, the erbium-doped fibre amplifier was introduced, which largely superseded periodic signal regeneration within unmitigated link distances. Optical amplifiers are particularly important for solitonic links (see below).

1.2 Major components of an Optical Communications System

A simple modern optical communications system normally contains a transmitter, optical fibre cable, a receiver, and amplifiers if the power balance condition is not satisfied without them.

In the simplest situation, the transmitter (commonly uses light-emitting diodes (LEDs) or laser diodes) initially sends information which is modulated into pulses carried by light; then the pulses go through the cable of optical fibre (which is usually made of high-quality silica glass); with the pulses decaying through the entire link, a number of amplifiers are needed due to re-enlarge the pulses in every certain distance; at last, a receiver is naturally necessary to get the pulses and decode to be the information sent by the transmitter.

1.3 Different Schemes of Modulations in Optic-Communications Systems

On-off keying (OOK) is detected directly; it is widely utilized for deployed fibre and free-space optical communication (FSO) systems.

Differential phase-shift keying (DPSK) with interferometric detection is being considered for those systems as well.

In order to increase the efficiency and capacity of spectrum, those techniques are both put forward as ideas for transmitting information in future communications systems, possibly in combination with wavelength-division multiplexing (WDM) technology.

An alternative way of encoding information, as discussed in more detail below, is pulse position modulation (PPM), which can be used in either fibre or FSO system. The important feature of such a system is that both the transmitter and the receiver must be synchronised to the same clock to ensure the information received correctly. Otherwise, the error probability is nearly one. Usually, this technique was analyzed as a sensitivity-constrained link.

The main features of the three schemes just mentioned are as follows.

i. **On-off keying**: the simplest form of digital modulation which also is a special form of amplitude-shift keying; simply switching on and off the carrier, a series of logical '1's and '0's will be represented. A binary '1's means that the carrier is 'on'; when the carrier is 'off', it shows a binary '0'.

ii. **Differential phase-shift keying**: a type of phase modulation that conveys data by changing the phase of the carrier wave. The receiver is important that been avoided for a coherent reference signal.

iii. **Pulse position modulation**: a pulse modulation technique which transmits signals by encoding M message bits using a single pulse time-shifted from a reference position by one of 2^M possible time-shifts. This technique will be discussed in more detail later as it is the main subject of this report.

The alternative modulation methods in optical communication are numerous; a large fraction of them are based on different methods of keying. However, in this project, I will only focus on on-off keying (OOK) and pulse position modulation (PPM), after all calculations, the comparison and conclusion will be given based on the numerical results.

1.4 Different media of optical communications

Optical communications fall into two categories as regards the transmission medium. These are free space optical communication and fibre optical communication. Both free-space optical and optical-fibre communication systems can use various modulation methods. Furthermore, a specific type of optical fibre communications is optical soliton communication. In this section, I introduce these communication systems.

1.4.1 Free-space optical communication

Free space optical communications is a kind of technology which transmitting signals only in free space. It is not like optical fibre communication, this scheme does not need any solid media as carrier, but only using light wave itself can transmit information.

Free space optical communications (FSO) is now normal for point to point communications between preset locations on land, and is also used for communication between moving platforms on land, on the surface of the sea, in air, and in space. Free space involves that it is not practical to use optical fibre to connect the points that need to correspond or exchange telemetry data [1].

Pulse position modulation (PPM), which is the subject of my project, is believed to be advantageous for very long (e.g. satellite) free space optical links because of its low average power requirements.

1.4.2 Optical-fibre communication

Optical fibre communication is the main technology in telecommunication industry. In recent decades, the aggregate bit rates achievable by fibre communications have grown to tens of TBit/s, covering all levels of optical communications from the rack-to-rack data links to backbone long-haul links of a length of tens of thousands of km.

As shown in *Figure 1* modern fibre-optic communication systems commonly include an optical transmitter to adapt an electrical signal into an optical signal to send into the optical fibre, a cable containing bundles of multiple optical fibres that is running scared through underground conduits and buildings, multiple kinds of amplifiers, and an optical receiver to claim the signal as an electrical signal. The information transmitted is typically digital information generated by computers, telephone systems, and cable television companies.

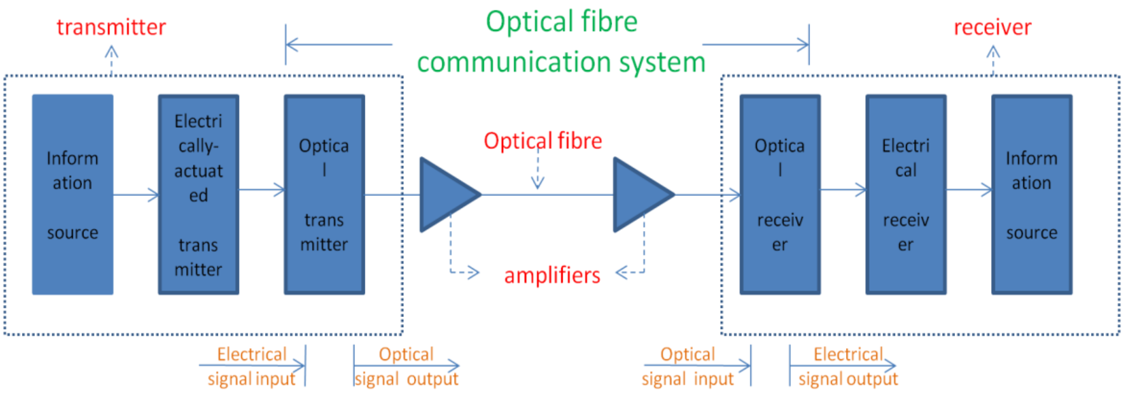

Figure 1 Optical fibre communication system

This technology has a large number of advantages. The frequency band of optical fibre is several orders of magnitude broader than that of coaxial cable, which leads to large information transmission allowance. Due to its low loss and low bit error ratio (BER), the amplifier distance could be extraordinary long. Comparing with electrical wave communications, the equipment of optical fibre communication are relatively low weight and small size. Because of optical fibre is made of glasses, it could save metals, in case to make the resources applies into other important area. And also it is safer to use this technology to transmit information rather than electrical wave communications.

There are several physical reasons which set the propagation limit in transmission links. In general, they can include loss, nonlinearity and dispersion.

i. Optical loss is due to the absorption and scattering in the fibre, combined with coupling loss and insertion loss of any components in the fibre channel. The longer the fibre, the greater the total loss, so in order to meet the power budget (power at the receiver exceeding its sensitivity), a link without amplifier can have a finite length. This is the sensitivity constraint in optical communications. It can also be seen as setting a limit to the minimum launched power.

ii. The fibre *nonlinearity* is a general term for all the effects that include the light intensity dependence of the refractive index of the fibre: self-and cross phase modulation, four wave mixing, nonlinear photon-phonon scattering etc. Self-phase modulation is the most fundamental of them. It leads to a pulse becoming chirped, with the sign of the chirp opposite at the leading and trailing edge. This, together with the dispersion (see below), leads to pulse distortion. Note that the non-linear effect grows with *intensity* (power per unit cross section area) rather than the power; the intensity and the area of the fibre core are inversely proportional, hence the effective core (or mode) area present in the mathematical analysis in the following chapters. The term *effective* cross-sectional area is usually used, since the power is not exactly uniformly distributed within the cross-section of the fibre [2].

Technically, the nonlinearity sets the upper limit of the power launched in the fibre, and for a given power value, limits the length.

iii. *Dispersion* is the phenomenon whereby the group velocity of the wave depends on its frequency [3]. It is determined by the structure and material of the media (fibre) we use. A modulated optical signal is by necessity broadened in frequency and has many frequency components. The dispersion makes these components propagate with different speeds, causing signal distortion.

The effects of dispersion and the nonlinearity are both the stronger the faster the variation of the signal amplitude with time. Therefore, they set a shorter propagation length limit for shorter pulses and therefore for higher bit rates. These two effects then determine the capacity constraint in communications.

In non-solitonic optical links, all three limitations are present. Solitonic links (see below) are a way of avoiding nonlinearity and dispersion, and facing only the loss limitation which is overcome by using amplifiers.

1.4.3 Soliton communication

For short, intense optical pulses, the balance between group velocity dispersion (GVD) and self-phase modulation (SPM) in the optical fibre can result in nonlinearity and dispersion cancelling each other. In this case, a soliton is created. It is a nonlinear pulse, in which the energy and duration are connected by a nonlinear relation, as explained later: the higher the energy, the shorter the pulse. In other words, solitons only keep their shape if the energy is constant.

Mathematically, the same approach describes two very different phenomena: spatial solitons and temporal solitons, though it is only the temporal solitons that are relevant to high speed communications.

The structure of soliton communication system (in the case of simple on-off keying, see below) is as follows:

i. The system requires an optical soliton source, which creates a number of extreme short pulses at regular intervals (often called optical soliton train or flow).

ii. A modulator superimposes some information on the soliton trains, suppressing pulses where an optical zero is needed and launching pulses that become optical ones into the fibre.

iii. A boosting amplifier can be used to compensate the losses in the modulator and to ensure the pulse energy is sufficient for them to evolve into solitons.

iv. As soon as these pulses are been modulated and enlarged, they are sent to fibre to transmit, and evolve into solitons as they propagate.

v. In case of energy loss in fibre, it is significant to put a number of Erbium-doped Optical Fibre Amplifiers (EDFA) on the fibre at fixed distance; which ensure soliton energy, and thus duration, remain constant on average. The loss in the fibre decreases energy, for solitons this leads to broadening, so has to be combated. Therefore, amplifiers are even more important in a solitonic link than in a non-solitonic one, and have to be closer spaced; otherwise, the energy loss will generate signal deterioration (broader solitons can cause intersymbol interference, or result in an increase in soliton interaction, introduced below).

vi. At the receiving end of the link, the receiver should contain a photon detector and a demodulator to restore the information.

The most important limitation to a soliton transmission link, at least at modest bit rates, is **Gordon-Haus jitter.**
The nature of this effect is as follows:

Compensation for fibre losses usually uses lumped amplifiers in a long-haul soliton communication system, as above. Amplifiers work through stimulated emission, which is a stochastic process, so the amplitude of the pulse at the output of the amplifier has some random variation, and so has the carrier frequency (chirp). Because the amplitude, chirp, timing, and duration are all interconnected in a nonlinear pulse, the random variation in amplitude leads to a random variation in timing. This random variation in timing is the Gordon-Haus Jitter. It was first proposed by J. P. Gordon and H. A. Haus in 1986 [5].

The statistics of the jitter because of intensive amplifiers adding into the spontaneous emission noise is Gaussian [4]; with a variance proportional to the cube of the total distance of the link has been shown by Gordon and Haus [5]. The formula for the variance has been derived in [5] and will be discussed in more detail in Chapter 2.

Soliton communications have been intensely studied over the last several decades. As an example, for direct connected optical fibre link, when the bit rate is 10 Gb/s, the transmission distance is up to 1,000 km; as soon as the bit rate increase to 20 Gb/s, the capacity of the transmission link can still have 350 km [6].

To increase the aggregate bit rates of soliton information transmission further, the technology of wavelength division multiplexing (WDM) can apply to optical soliton systems. Therefore, this technique could be widely used in the future.

1.5 Soliton Interaction in Fibres

When two solitons in the fibre are close to each other – which may happen in high bit rate optical systems - they start interacting, leading to the separation between solitons changing with transmission distance, either monotonically or non-monotonically.

In such soliton-soliton interactions, it is known that the relative phase and the initial separation play a role in the outcome of the interaction. Soliton interactions are sometimes described by means of interaction forces. These forces are proportional to the sine of the phase difference between the solitons, so that the solitons with the same phase attract and solitons with the opposite phase repulse, like charged particles. The interaction forces decrease exponentially with the time separation between the solitons, as will be described in Chapter 2. As a result, when two or more solitons are very near, normally if the interval is less than a bit slot, they would interact each other. As a result, one soliton in the front may come a little late, and another soliton in the back probably comes a bit early. This will deteriorate the performance of a link and will be one of the subjects of this study.

1.6 Report Structure

The report structure is as follows. Chapter 2 contains the overview of the prior work relevant to the project. Chapter 3 introduces the theoretical formalism including the introduction of the objectives and the structure of the system models of the project. In Chapter 4, the main results of the calculations will be presented. Chapter 5 briefly considers the applicability of PPM as opposed to OOK in non-soliton RZ communications. Chapter 6 will give some possible considerations of implementations of PPM in high bit rate. The conclusions and discussion of possible future work are discussed in Chapter 7.

Chapter 2 Literature review

The project brought together two issues that were, in earlier literature, investigated independently: the comparison of PPM and OOK modulation methods in communications and the issue of soliton interaction. Therefore, the literature review will consist of two parts. Firstly, I shall review the main characteristics of pulse position modulation as compared to on-off keying, and then introduce the prior work on soliton interaction.

2.1 Introduction of main characteristics over on-off keying and pulse position modulation

In this part, I will introduce the main characteristics of on-off keying and pulse position modulation schemes in high bit rate optical communications, how they work, and their advantages. Some current applications will also be discussed.

2.1.1 On-off keyed link under consideration

A simple model of pulses in an on-off keying (OOK) link is shown in *Figure 2*.

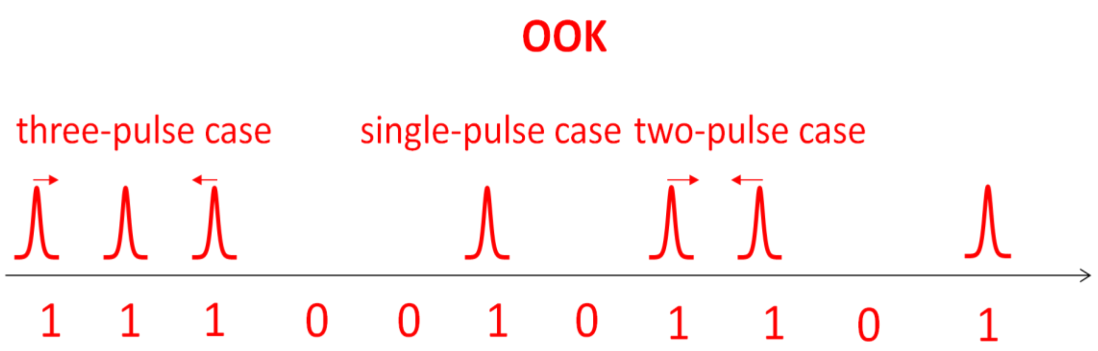

Figure 2 Schematic structure of a fragment of a pulse sequence in an OOK link

As seen in the figure, the binary ‘1’s are denoted by pulses which carry information, whereas the ‘0’s are slots without a pulse. This technique transmits signals by using a key located in the transmitter part and acting on a stream of short pulses. When the key is ‘on’ during a bit period, the transmitter sends a pulse which becomes a logical ‘1’s; as soon as the key is ‘off’, there is no signal sending, which shows as a ‘0’ during the corresponding bit. Note that the pulse sequence in a pseudorandom sequence of bits can include a single pulse (logical one) surrounded by zeros, or two logical ones in a row or 3+ logical ones. In the case of a soliton link, this will be important for the analysis of pulse interaction which will be presented below.

Using OOK to transmit information, the very first advantage is that the OOK technique only needs the simplest equipment configuration. Secondly, it is more efficient compared to frequency-shift keying (FSK), at the expense of the somewhat higher sensitivity to noise [7].

In modern high bit communications, it is still widely used for transmitting, say, Morse code. In addition, radio frequency (RF) carrier waves would also like to use this technique due to the low requirement of equipment which means low cost. Sometimes it is also being used in principle digital encoding scheme, particular computing band, maybe airport broadcast and even signal lights control.

2.1.2 Pulse position modulation link under consideration

Unlike the OOK modulation, in which a single pulse or pulse-free slot is attributed to a single bit of information, PPM can encode M>1 bits in a single *pulse frame* (M=1 is also possible but not often used). The parameter M is known as the *order* of the PPM scheme and is very important for its operation. M message bits are then encoded together by transmitting a single pulse in one of 2^M possible time-slots, which together form the frame. Each frame also can include a *guard band* to minimise inter symbol interference (essentially, the soliton interaction in our case). The duration of the frame is $T_{frame}= M/B$, the duration of the guard band is by definition $T_{guard}= (1\text{-}m)T_{frame}$, where the parameter m ($0<m\leq 1$) is known as the *modulation index* of the PPM scheme, and the duration of each slot is thus $T_{slot}= mT_{frame}/2^{M}= mM/(2^{M}B)$.

Figure 3 shows a fragment of a pulse sequence in a PPM transmission link.

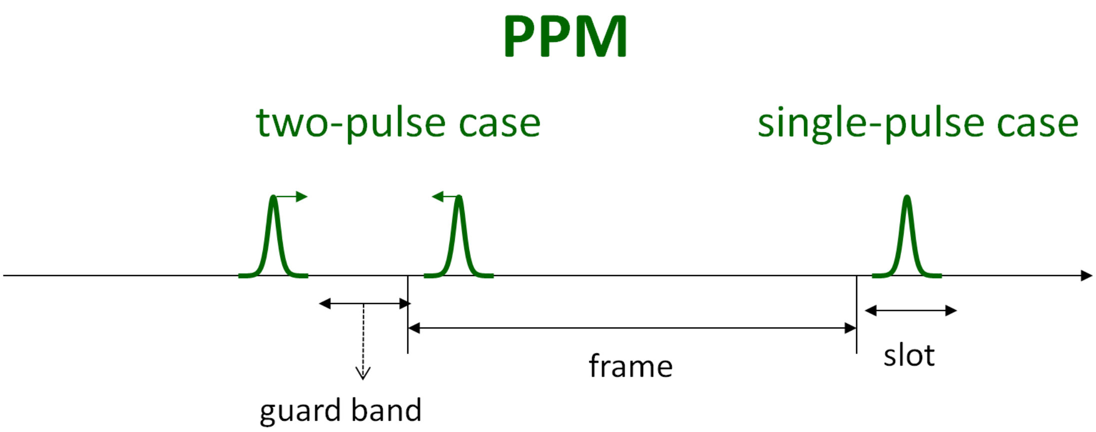

Figure 3 Schematic structure of a fragment of a pulse sequence in a PPM link

This technique can be used for a number of purposes; the very best area is still RF communications, possibly including RF over fibre. Due to the simplicity the technique is, the low requirement of equipment commonly is lightweight. As a result, it is normally applies in wireless remote controlling. The sensitivity makes the scheme low efficiency; however, for the applications recently, it is much safe to use this in applications such as electrical lock [8].

2.2 Solitons in pulse position modulation

In this section, I will concentrate on solitons in PPM, which until now have been analysed in the absence of soliton interaction, and the comparison of PPM and OOK performed by previous authors.

Pulse position modulation (PPM) is been demonstrated as an effective modulation format for use in the optical-fibre link [9]. Due to the fact that PPM transmits information by the discrete temporal slot position of the pulse in a time frame, timing synchronization becomes far more important for PPM systems over OOK (the authors of [9], [10], [11] called it PCM) systems [11]. As they put it, ‘Signal blocks which each contain a single optical pulse are transmitted successively and the information is conveyed by the pulse position within each block. Pulsing the laser allows average power to be converted into narrow, highly peaked pulses which aid detection.’[10]. In [10] the authors examined an optical-fibre soliton system employing the PPM modulation format and compare the fundamental limit on the system length to an OOK optical soliton system. In the absence of soliton interaction, the main impairment to soliton links is the Gordon Haus jitter as discussed in Chapter 1, which is by definition ‘the jitter brought about in soliton arrival time by the subtle change in central frequency of the soliton when amplified spontaneous emission (ASE) noise from the in-line optical amplifiers becomes assimilated into the soliton pulse’ [10]. Quantitatively, it is characterised by mean pulse position deviation σ calculated from the relation [4]

$$\sigma^2 = \frac{n_{sp} n_2 D h (G-1) L^3}{9 T_s A_{eff} L_{amp} Q} \tag{1}$$

$$Q = \ln \frac{G}{1 - 1/G} \tag{2}$$

where all the parameters have their standard values: L is the propagation distance, n_{sp} and G are the population inversion factor and the gain of each of the (identical) in-line amplifiers that maintain the average soliton energy constant, T_s is the soliton duration, D is the dispersion of the fibre, h is the Planck constant, A_{eff} is the effective fibre mode cross-section, L_{amp} is the distance between the amplifiers (so that in the case of average power maintained constant, G/L_{amp} is approximately the fibre loss per km), and Q is the power-enhancement quality factor.

The length of the link is then derived as

$$L = \left[\frac{9 T_s A_{eff} L_{amp} Q}{\sigma^2 n_{sp} n_2 D h (G-1)} \right]^{1/3} \qquad (3)$$

, where the σ is calculated from the condition of BER = 10^{-9}:

$$\sigma = t_w / \sqrt{2} \mathrm{erfc}^{-1}(10^{-9}) \qquad (4)$$

The authors of [10] considered two different ways for comparing the Gordon-Haus effect in the soliton regime of PPM to OOK. First of all, assuming exactly the same soliton pulses are used, from the same (jitter-free) laser source, hence with the same amplitude and duration, carrying the same information, for both modulation formats. The amplifier separation and other properties of the link are also assumed to be common for the two modulation schemes. In such a comparison, PPM is enabled to retain its usual benefits in terms of lower average power (with only one pulse used to denote several bits). However, as pointed out in [10], PPM under such terms is more vulnerable to the Gordon-Haus effect due to the smaller time window in which the PPM soliton is required to remain to avoid an error. As an alternative, the authors considered the case of average power carrying the same information for both systems. This meant ‘trading off PPM’s customary advantage of lower average power for possible reduced vulnerability to the Gordon-Haus effect’. The use of considerably sharper, higher amplitude pulses for the PPM regime was necessary in such case. The authors realized however that the possibility is not advantageous and even more vulnerable to the Gordon-Haus effect under such terms than with identical pulses [10].

The general conclusion made by the authors of [10] was that PPM was more vulnerable to jitter in the pulse arrival time due to the Gordon- Haus effect under all conditions they studied. They found the best system length achievable with PPM to be at very best 65% of a comparable OOK system, and in many cases even shorter [10].

2.3 Soliton interaction and its effect on soliton communications

In this section, I would like to overview soliton interaction and its effect on soliton communications as considered in the previous work – which has been done so far only for OOK links.

The general character of soliton interaction was demonstrated in the section 1.5. The effect of soliton interaction in soliton communications becomes very significant when the spacing between the solitons becomes comparable to the pulse duration. In a soliton communication system, the soliton interaction effect needs to be combined with that from Gordon-Haus jitter (GHJ) [12], [13]. At low bit rates, the soliton interaction can be neglected leaving GHJ as the sole source of error, but in soliton communication systems working at high bit rate, soliton interaction becomes a phenomenon which can be more important than GHJ. To the best of our knowledge, a semi-analytical as well as numerical analysis of the soliton interaction effect on communications was performed only in one paper [4], which also summarized the previous work and which we shall follow here.

To study multiple-pulse systems, the authors of [4] start with the nonlinear Schrödinger Equation (NLSE) for the case of two solitons a small time away from each other. They solved this both numerically and quasi-analytically, by obtaining analytical estimates to the pulse distance due to interaction, and comparing numerical and analytical results.

An approximation for the case of two solitons based on treating solitons as quasiparticles exerting forces on each other had been previously presented in a laregely mathematical paper by Karpman and Solov'ev [14]. Gordon (the co-discoverer of GHJ) had also himself derived another approximation directly from the exact two-soliton equation [15]. In dimensionless soliton units (see Appendix 1), the expression for the dimensionless amplitude of the field in a soliton pair u has the general form [4]

$$\begin{aligned} u(\tau,q) = \exp(i\Omega)\{ & A_1 \operatorname{sech}\left[A_1(\tau - q)\right]\exp(i\theta_1) \\ & + A_2 \operatorname{sech}\left[A_2(\tau + q)\right]\exp(i\theta_2)\} \end{aligned} \tag{5}$$

The expression for q is in general given by a complex transcendental equation [4].

The authors of [4] considered the expression above for the case of a zero phase difference between neighbouring solitons which is known to lead to the worst case interaction. A simpler expression for the separation between the two solitons is then obtained as a particular case of the general one [4]:

$$q = q_0 + \ln[\cos(a\zeta)] \tag{6}$$

, where

$$a = 2\exp(-q_0) \tag{7}$$

, q_0 is the initial separation; ζ is propagation distance, both in normalized solitonic variables.

The next step was extending the above results to the case of three solitons in a row, rather than two. The authors of [4] observed that the force between adjacent solitons depended in general mainly on their separation (and relative phases, but in the analysis these were set to be equal). Then in a system of three solitons where the distance and relative phases between side solitons (*Figure 2*, the first and last pulses of three-soliton case) and the middle soliton (the middle pulse of three-soliton case) are the same, the interaction of solitons on both sides of three-soliton case relative to the middle one, are of opposing signs. Because of these balancing forces, the middle soliton is fixed and the two outer solitons interact as in the two-soliton case, but with a longer distance between them [4]. Indeed, the separation in the two-soliton case is equal to 2q. However, in the three-soliton case, because the middle soliton is fixed, it the interaction between the outer two solitons that matters. The separation between each of the side solitons and the central one is q_0+q, where q_0 is the initial separation. For the value of q, the expression was given [1]:

$$q = q_0 + \ln[\cos^2(\frac{1}{\sqrt{2}}a\zeta)] \tag{8}$$

Where a is still given by (7).

In order to generalize the results even further, starting with the four-soliton case would be necessary. In that case, the two middle solitons are practically fixed, and each one is surrounded by neighbour pulses exerting, as in the three-soliton case, forces in opposite directions. With a good degree of approximation, assuming that the two middle solitons are fixed and the solitons in two sides behave in a similar way to the three-soliton system, but the time interval between them is larger. Then, because the interaction forces are known to decrease very fast (exponentially) with the distance between the interacting solitons, the authors concluded that this case is essentially equivalent to that of a single soliton – an approximation we shall follow in our paper.

The comparison of time between solitons calculated analytically with numerical simulations was given by in *Figure 4*.

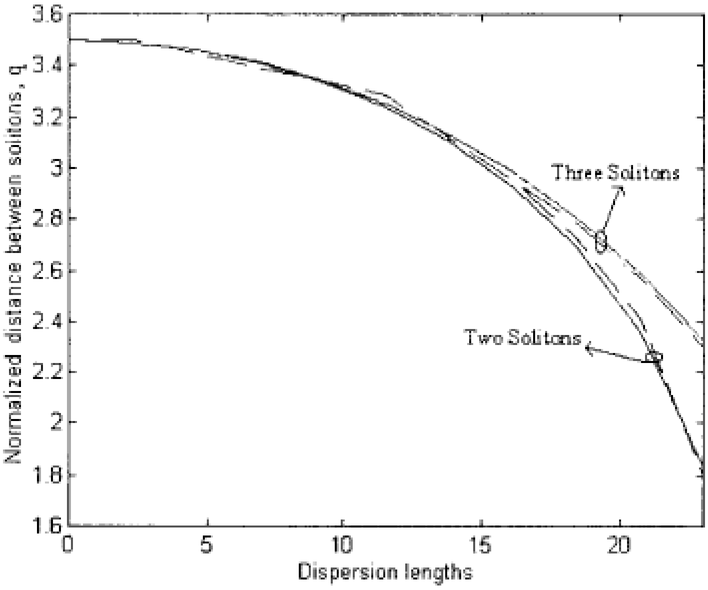

Figure 4 Normalized distance between solitons. The solid line is the analytical solution, the dashed line is obtained by numerical solution of the NLSE. [4]

The figure shows an excellent agreement, so we concluded that we could use the analytical theory developed by the authors of [4] in our simulations, rather than performing first principles numerical analysis.

The authors of [4] consider a random binary sequence with 2^5-1 bits that contains all possible sequences of 5 bits [16]. Then the analytically calculated correction to the pulse position in a stream as a function of *L* is determined, in the two or three pulse case and in dimensional parameters, as:

$$t_2 = T_s \left| \ln \left| \cos\left(\frac{aL}{L_D} \right) \right| \right| \quad (9)$$

$$t_3 = 2T_s \left| \ln \left| \cos\left(\frac{aL}{\sqrt{2}L_D} \right) \right| \right| \quad (10)$$

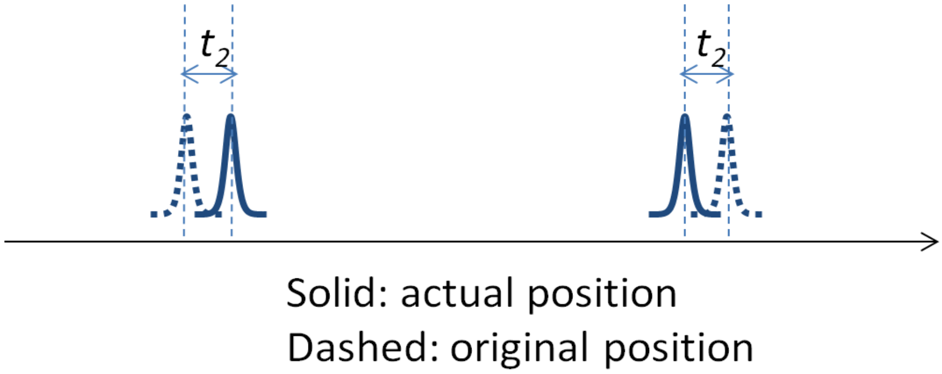

Figure 5 Pulse position evolution in the two-soliton case

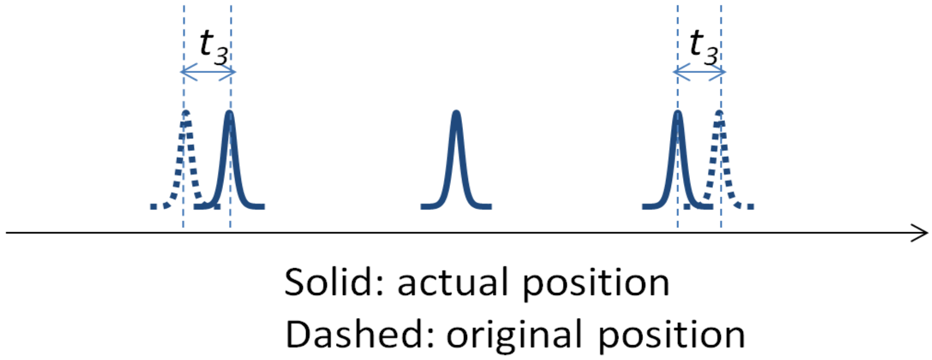

Figure 6 Pulse position evolution in the three-soliton case

Figure 5 illustrates the evolution of soliton spacing, with $2t_2$ being the propagation distance -dependent correction (shortening) of the separation between solitons for two-soliton case; t_3 is the corresponding parameter for the three-soliton case (shown as *Figure 6*); T_s stands for the soliton duration, L is the propagation length (which is the most important parameter in this paper), $L_D = T_S^2 / |\beta_2|$ is the dispersion length, $\beta_2 = -\frac{\lambda^2}{2\pi c} D$ being the second order group velocity dispersion parameter (GVP) of the fibre, and a is given by (7).

The formulas above are for generic case of two interacting solitons. Specifically in the case of OOK digital communication system, the initial separation is simply given by the duration of the bit slot. Then, using the usual dimensional variables (as opposed to normalised solitonic parameters) we obtain

$$a_{OOK} = 2\exp\left(-\frac{1}{2BT_s}\right) \tag{11}$$

Here, B is the bit rate of the signal. Correspondingly, $1/B$ is the bit period, or the distance between the notional positions of the two pulses in the stream.

Two solitons (logical ones) in two adjacent slots get attracted or repulsed depending on their relative phase. As shown above, maximum change (shortening) of the interaction interval occurs in the worst case of equal phases and is $2t_2$, where

$$t_2^{(OOK)} = T_s \left|\ln\left|\cos\left(\frac{a_{OOK}L}{L_D}\right)\right|\right| \tag{12}$$

, which is essentially Equation (9) for the case of an OOK stream, with the parameter a_{OOK} determined in (11).

In the case of three solitons (logical ones) in a row, as shown in *Figure 2*, the middle pulse stays in its targeted location, but the two outer ones get (in the case of equal phases) attracted where as discussed above (Equation (10)):

$$t_3^{(OOK)} = 2T_s \left|\ln\left|\cos\left(\frac{a_{OOK}L}{\sqrt{2}L_D}\right)\right|\right| \tag{13}$$

This analysis for the two-and three-pulse case allowed the authors of [1] to obtain an analytical estimate for the arrival time distribution of a soliton stream. In the absence of jitter, this distribution consists of infinitely thin lines (delta functions) caused by the cases of a single-pulse, two-pulse, three-pulse, and multiple pulses in a row. The amplitudes of these lines are related to the probabilities of the corresponding case happening in a long pseudorandom bit stream.

The case of a single pulse obviously gives rise to a line in the centre of the distribution (t'=0, where t' is the relative arrival time, counted from the centre of the bit).

The authors of [4] estimated that in a random stream, the probability of the two-pulse case occurring is 0.25, and the same for the three-pulse case. The rest is either single-pulse case, or multiple pulses in a row (more than three).

In the two-pulse case, the centre of gravity of the two solitons does not move, since the deviation can be positive or negative. so each of the pulses shifts symmetrically by $t_2^{(OOK)}$ As a consequence, the 25%=0.25 probability of the two soliton case gives rise to two lines, with an amplitude of 0.125 each, on the two sides of the central probability lobe, centred at $t'=$ $t_2^{(OOK)}$ and $t'=$ - $t_2^{(OOK)}$.

In the three-pulse case, the pulses also shift symmetrically, as in the case of two pulses, giving two extra lines in $t'=$ $t_3^{(OOK)}$ and $t'=-$ $t_3^{(OOK)}$ in the three pulses case. As in the two-pulse cases, the 25% probability of the three-pulse case leads to two lines at an amplitude of 0.125 on both sides of the central maximum.
Since the distance between the interacting solitons is longer in the case of three pulses, $t_3^{(OOK)} < t_2^{(OOK)}$ and, as discussed above, streams of more than three solitons do not need to be included separately – they are effectively equivalent to the single pulse case. Then, the amplitude of the central line at t'=0 is 1-0.25-0.25=0.5

In order to check their analytical findings of the effect of soliton interaction on the arrival time distribution, the authors of [4] first discounted the GHJ and checked the validity of the analytical result by repeating the 20 Gb/s OOK system simulation with an ideal amplifier (n_{sp} = 0). The total length of the link was chosen to be nine dispersion lengths.

The arrival time was found to consist of a line given by the single pulse, or a logical one surrounded by logical zeros on either side, which arrives on average at its targeted value with 50% weight for zero-time deviation (a Gaussian centred at zero); 25%

corresponding to a deviation equal to the two-soliton case, and 25% with a deviation equal of the three-soliton case (each split into two symmetric lines of 12.5%), in excellent agreement with the analytical theory.

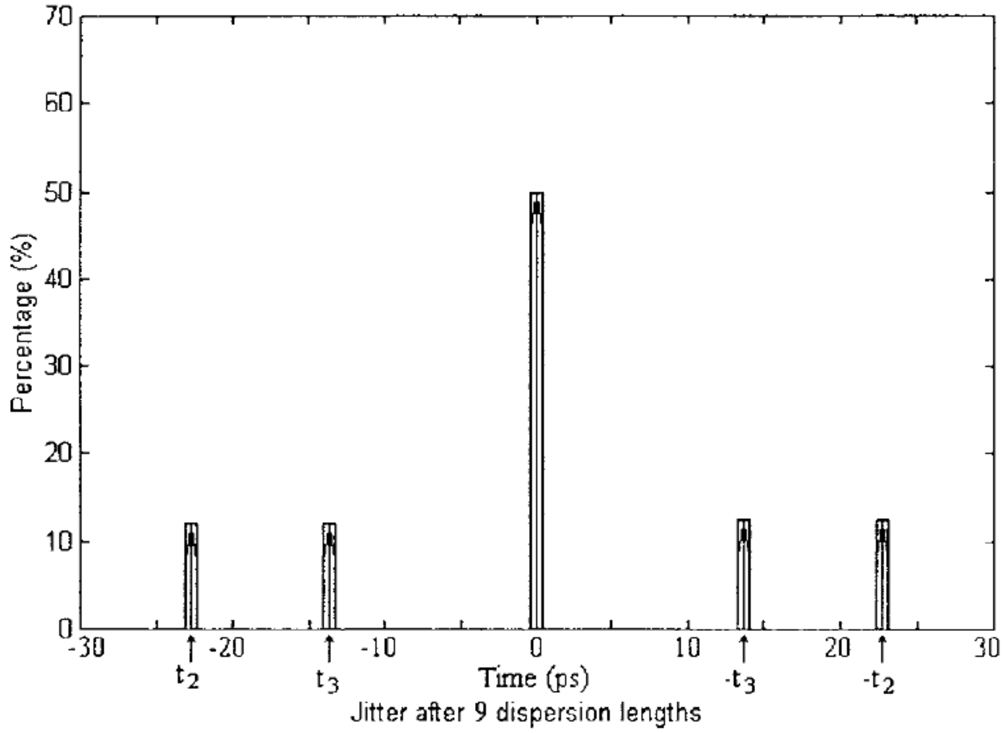

Figure 7 Arrival time statistics of a pseudorandom sequence, after the propagation of nine dispersion lengths. The bars were obtained by numerical simulation. [4]

The analytical distribution expression corresponding to *Figure 7* is given by [1]:

$$p(\mathrm{t}) = \frac{1}{2}\delta(\mathrm{t}) + \frac{1}{8}\left(\delta(\mathrm{t}-t_2) + \delta(\mathrm{t}+t_2) + \delta(\mathrm{t}-t_3) + \delta(\mathrm{t}+t_3)\right) \qquad (14)$$

, where in the case of no GHJ, the width of the delta-function is only given by numerical constraints.

Figure 7 also illustrates the excellent agreement between the analytical results and the numerical simulations. The finite numerical resolution (resolution = 0.78 ps) produces a broadening in the delta function; however, the position of the side bars are exactly the positions predicted by the analytical theory, and so are their relative magnitudes – they are to a very high accuracy given by (14).

The noise which is at the outside of the bandwidth of the optical signal can be removed, using an appropriate optical filter, leaving only the weakened in-band noise, which is unavoidable and co-propagates with the signal.

The co-propagation of signal and noise changes the arriving time statistics presented in expression (14) and thus degrades the signal noise ratio (SNR). Expression (14) is not valid any more, for the reason of the Gordon–Haus effect being present in the system. However, the authors of [4] found that it required only a minor modification: changing the delta-functions to broadened delta-functions. Indeed, in a soliton communication system, the noise is only a small correction to the signal power, so one can assume that the noise only produces a little change in the phase and amplitude of each pulse. Then, the authors of [4] made a very reasonable assumption that the jitter *pdf* in a system with noise would be related to the one presented by the formula (14). However, if one reinstalls the GHJ and combines it with pulse interaction then, instead of five discrete lines shown in the formula (14), one ends up with a combination of five broadened (Gaussian) lobes, with the centre of each Gaussian determined in the same way as in (14):

$$p(t',L) = \frac{1}{2} f_\sigma(t') + \frac{1}{8}\left(f_\sigma(t'-t_2^{(OOK)}) + f_\sigma(t'+t_2^{(OOK)}) + f_\sigma(t'-t_3^{(OOK)}) + f_\sigma(t'+t_3^{(OOK)}) \right) \tag{15}$$

Note that in this curve, the dependence on L is implicit through both Gordon-Haus jitter (16) (GHJ, qualified by parameter σ) and soliton interaction ($t_2^{(OOK)}$ and $t_3^{(OOK)}$). As discussed above, it is assumed that the shape of the GHJ broadened line is a Gaussian curve

$$f_\sigma(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{t^2}{2\sigma^2}\right) \tag{16}$$

, with the width σ was given by (1), which substitutes the simple delta-function used in (14).

The authors of [4] did not translate their calculations of arrival time distribution of the solitons into a calculation of the maximum propagation length, nor the dependence of soliton duration to the link. This is one of the objectives of our work. Also, they only worked with OOK; our objective is to extend their analysis of soliton interaction effect in communications to PPM links.

Chapter 3 Theoretical formalism

This chapter introduces the theoretical formalism used in our work, including the introduction of the objectives and the structure of the system models of the project.

➢ Theoretical model

We use the theoretical model of soliton interaction link which was derived by A. Pinto and G.P.Agrawal [4], combined with the PPM analysis such as the one presented in [10]. For the purpose of making the comparison, we use the same initial power to launch a number of pulses into both solitonic links which are using on-off keying (OOK) and pulse position modulation (PPM).

Then analytical approach was used that standard general definition of the link length limit L_{max} imposed by the arrival time distribution is

$$BER\left(L_{max}\right) = 10^{-9} \tag{17}$$

, taking 10^{-9} to be the standard of error-free operation, as in earlier papers which assumed the requirement of described no more than 10^{-9} for optical fibre link. It is assumed that the bit error rate (BER) is only determined by the link (any additional errors introduced by the receiver are negligibly small), and so the BER is given

$$BER(L) = \int_{-\infty}^{-t_w} p(t',L)dt' + \int_{t_w}^{\infty} p(t',L)dt' \tag{18}$$

Here, as in the previous chapter, the blind variable *t'* is essentially the deviation of the arrival time of the pulse from its targeted (ideal) value at the centre of the bit period (slot), $2t_w$ is the timing window for each of the modulation schemes, and p is the distribution of pulses in their arrival time.

The function p (which in an ideal case of no jitter and no pulse interaction would be a delta-function) is dependent on the propagation distance L since both the Gordon-Haus jitter (GHJ) and the soliton interaction are functions of L; however, the exact feature of the dependence for the different modulation schemes we consider is, obviously, different.

Next, I divide the models into two cases: OOK and PPM.

3.1 On-off keying model

In the case of on-off keying (OOK, which the authors of [9], [10], [11] call pulse code modulation, PCM), the nature of the arrival time distribution function p is given directly by the equation (15) as described in the previous section.

The mechanism of GHJ broadening of f (shown as expression (16)) alone (with $t'=t$) would be consistent to the case treated in [10].

In *Figure 8*, we have an example of a fragment of a pseudorandom OOK bit sequence. It includes a sequence of 10 bits that we can reduce to the cases of isolated pulse (single-pulse case, which is shown as the sixth bit), two-soliton case (shows as the pulses at the eighth and ninth bits) and three-soliton system (the three pulses at the beginning). In order to analytically establish the arrival time figures, it is enough to consider at most three neighbour solitons because the interaction forces reduce exponentially with solitons separation and so four and more solitons in a row are equivalent to the single pulse case as discussed above. We consider a random binary sequence with bits that contains all possible sequences of 5 bits [11].

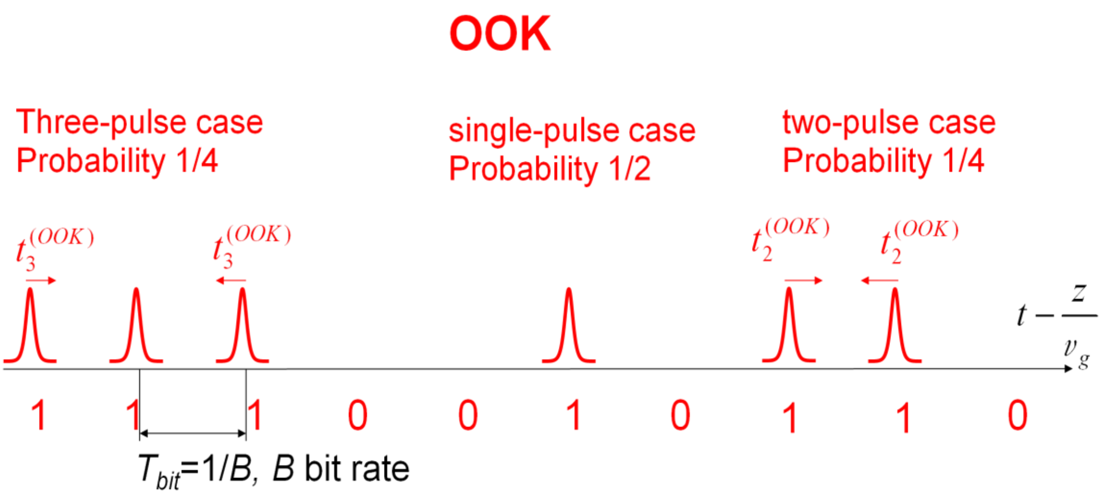

Figure 8 A pulse sequence of generic on-off keyed link in details.

To illustrate the probability of arrival time directly, the distribution is given as Figure 9.

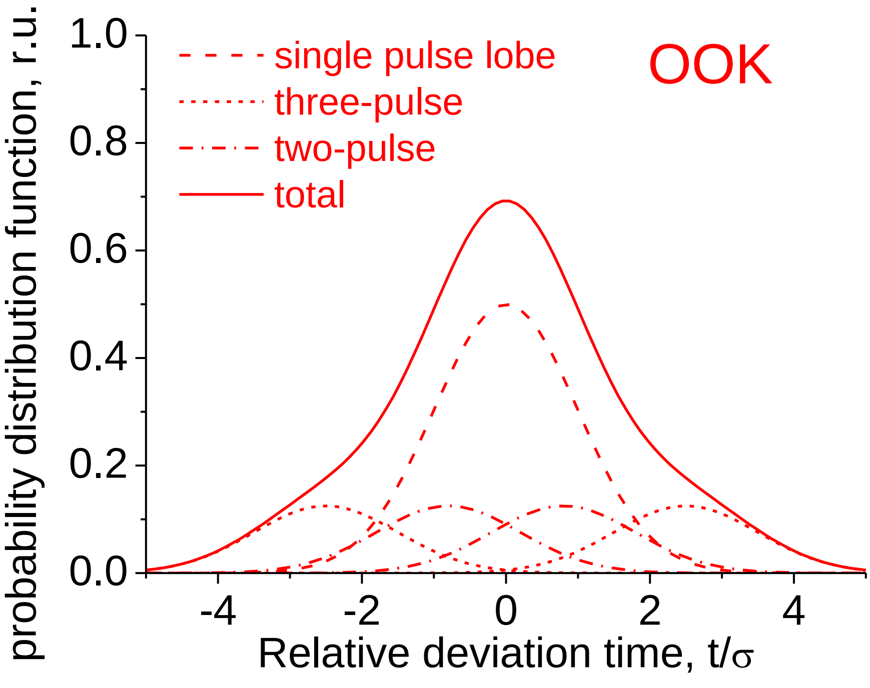

Figure 9 Pulse arrival time for on-off keying

The authors of [4] did not explicitly write out the expression for the bit error rate; however, since we have equation (15), it is easy to calculate the BER as

$$BER_{OOK}(L)=\frac{1}{2}\mathrm{erfc}\left(\frac{t_w}{\sqrt{2}\sigma}\right)+$$
$$\frac{1}{8}\left(\mathrm{erfc}\left(\frac{t_w-t_2^{(OOK)}}{\sqrt{2}\sigma}\right)+\mathrm{erfc}\left(\frac{t_w+t_2^{(OOK)}}{\sqrt{2}\sigma}\right)+\mathrm{erfc}\left(\frac{t_w-t_3^{(OOK)}}{\sqrt{2}\sigma}\right)+\mathrm{erfc}\left(\frac{t_w+t_3^{(OOK)}}{\sqrt{2}\sigma}\right)\right) \quad (19)$$

The propagation length L_{max} can then be calculated by substituting (19) into (17) and solving the resulting transcendental equation.

3.2 Pulse position modulation model

As mentioned before, there are *M* message slots in a pulse position modulation transmission frame. If *M*=1, there will be only two slots and one guard band in each frame (shown as *Figure 10*); it seems like a special OOK link which just has single-pulse case and two-soliton case. The pulse in each frame will be in either the position ‘0’ or the position ‘1’.

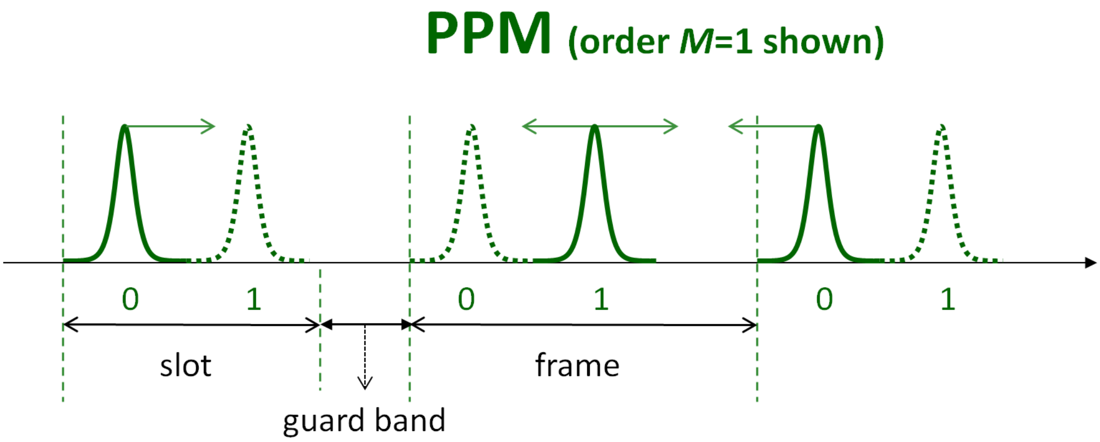

Figure 10 Schematic structure of a fragment of a pulse sequence in an order M= 1 PPM link

In Chapter 4, I will briefly consider the fact of *M*= 1 and compare this situation with the object situation which is *M*= 2.

If M becomes relatively large, although there is low soliton interaction, the dispersion starts to be important (shown as *Figure 11*).

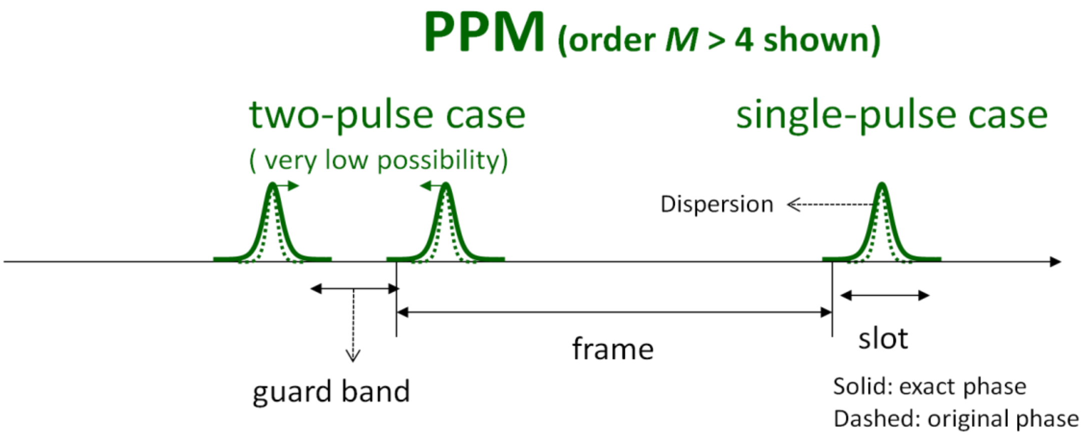

Figure 11 Schematic structure of a fragment of a pulse sequence in an order M> 4 PPM link

Thus, in previous papers [9], [10], [11] that studied the effect of GHJ on soliton links in the absence of soliton interaction, it was found that M=2 gives the longest propagation distance for a given soliton energy. Therefore, following [9], [10], [11], I will concentrate on the specific situation with the PPM order M=2. In such a case, there are 2^2 =4 slots and a guard band in each frame. The pulse in each of the slots represents two bits at once, for example 00, 01, 10, 11 (the order is determined by the system designer).

The difference between the PPM and the OOK from the soliton interaction point of view is that in the PPM modulation scheme, having three closely spaced pulses is impossible: this technique only can have either the two pulse situation or the single pulse situation.

The first case is shown is that of the first two pulses in *Figure 12*. The pulses are relatively closely spaced pulses, which will only occur in the case when one frame has a pulse in the last of its slots and the subsequent frame has a pulse in the first of its slots;

only the guard band and one slot duration are in between them, which is the only possibility when we could have soliton interaction. Both of these (the pulse in the last slot of the frame and another pulse in the first slot of the following frame) happen with a probability $1/2^M$ in a pseudorandom bit stream. Hence the probability of this two-pulse situation is

$$p_2 = 1/2^{2M} \tag{20}$$

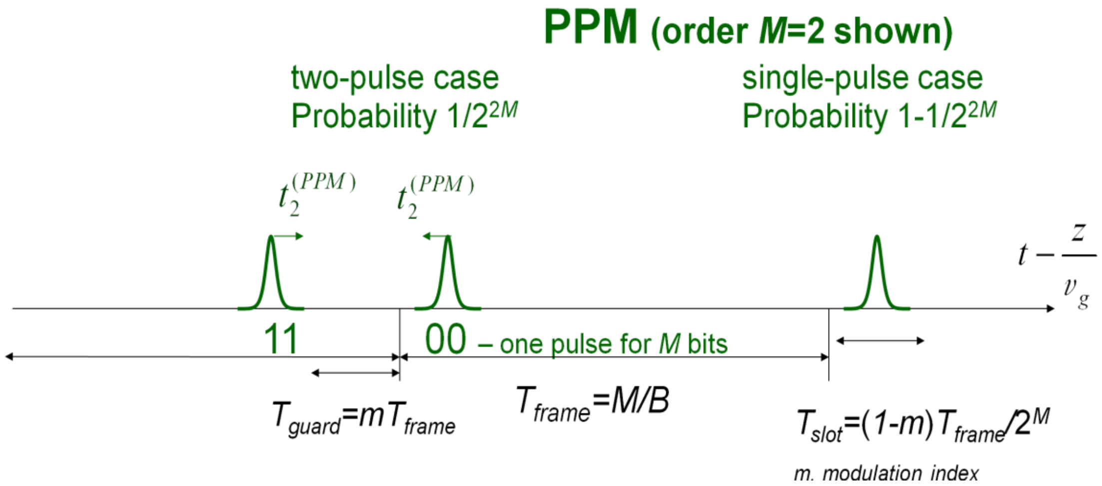

Figure 12 A fragment of a pulse sequence of pulse position modulation link in details with the order M= 2.

The second situation is when the solitons are far away from each other (more than two slots). This happens in all cases (like the two last pulses shown in *Figure 12*) except when one soliton is the last slot in the frame, and the other is the first slot in the next frame. This situation happens most frequently, with the probability

$$p_1 = 1-1/2^{2M} \tag{21}$$

Figure 12 presents the schematic time intervals of pulse position modulation for the case of M=2. The distance separating the pulses in the two-pulse case is $T_{slot}+T_{guard}$, as opposed to $1/B$ in the case of an OOK stream.

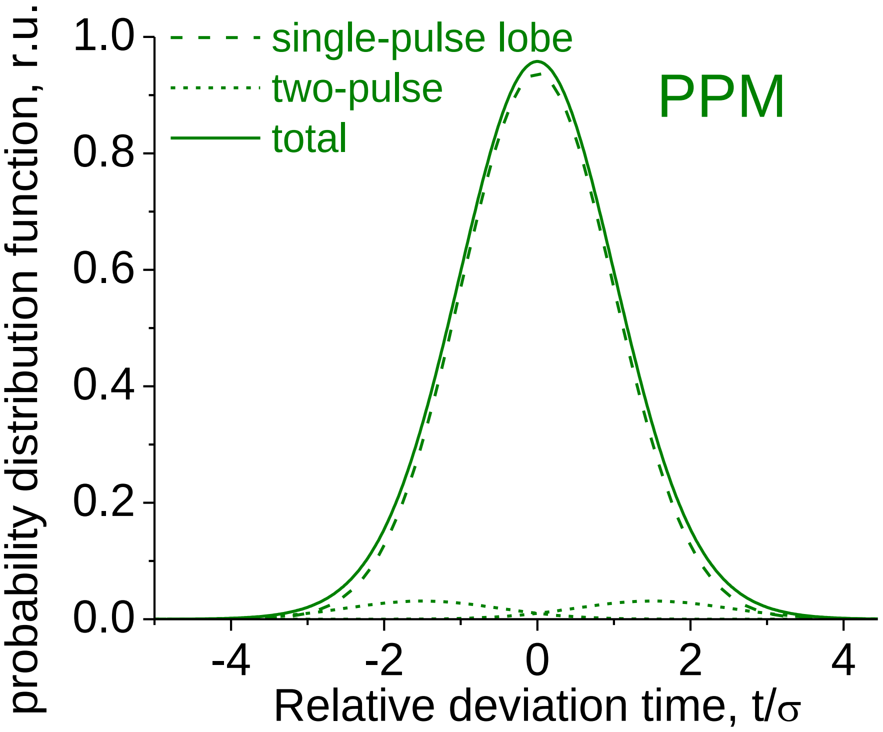

Figure 13 Pulse arrival time for pulse position modulation

As illustrated in *Figure 13*, the pulse arrival time probability density function (*pdf*) in the PPM scheme becomes a three-lobe curve is in the form

$$f(t') = p_1 f(t') + \frac{p_2}{2} f(t'+t_2) + \frac{p_2}{2} f(t'-t_2) \qquad (22)$$

, where the reason for the split of the two-pulse contribution into two symmetric lines in the expression (14), is the same as in the expression (22). The function f would be a delta-function in the absence of Gordon-Haus jitter, but we know in advance that the GHJ would be important so need to include it. Hence each of the lines is taken to be Gaussian broadened (f=fσ) as in Equation (16) and so the arrival time distribution in the case of a PPM link is given by:

$$f(t',L) = \left(1 - \frac{1}{2^{2M}}\right) f_\sigma(t') + \frac{1}{2^{2M+1}}\left(f_\sigma(t'-t_2^{(PPM)}) + f_\sigma(t'+t_2^{(PPM)})\right) \qquad (23)$$

, where, as in (9),

$$t_2^{(PPM)} = T_s \left| \ln \left| \cos\left(\frac{a_{PPM} L}{L_D} \right) \right| \right| \tag{24}$$

And, similarly (but not identically) to (11),

$$a_{PPM} = 2\exp\left(-\frac{T_{slot} + T_{guard}}{2T_s} \right) \tag{25}$$

Here, the distance between the pulses has been adjusted for the PPM case.

Thus, the BER for the PPM scheme is given by

$$BER_{PPM}(L) = \left(1 - \frac{1}{2^{2M}}\right) \mathrm{erfc}\left(\frac{t_w}{\sqrt{2}\sigma} \right) + \frac{1}{2^{2M+1}} \left(\mathrm{erfc}\left(\frac{t_w - t_2^{(PPM)}}{\sqrt{2}\sigma} \right) + \mathrm{erfc}\left(\frac{t_w + t_2^{(PPM)}}{\sqrt{2}\sigma} \right) \right) \tag{26}$$

The expression (26), like (19), contains an implicit dependence on L through both σ and $t_2^{(PPM)}$ and can be substituted into (17) for finding L_{max}. Following [10], we set t_w =0.3/B for the OOK case and t_w=0.5T_{slot} for the PPM case.

Chapter 4 Results

To make sure every issue which affect the solitonic link studied, we need first to research the bit rate and soliton duration dependence of the maximum transmission distance of the link. , making sure the effects of both the soliton interaction and GHJ are taken into account consistently. Since the results are strongly dependent on the parameter values used, the parameters should be introduced at the very beginning.

4.1 Final Specifications

As mentioned before, to make sure the two schemes are compared for a similar situation, we decided to use the same initial power and also generally the same parameters describing the main physical effects and properties of the link. The parameters are listed in *Table 1*.

Parameter	Final Specification
Separation between amplifiers	30 km
Wavelength	1550 nm
Attenuation	0.2 dB/km
Nonlinear coefficient	3×10^{-20}
Effective mode area	10 μm^2
Dispersion	0.1 ps/(nm · km)
Spontaneous emission factor	2.0
Standard bit error ratio	10^{-9}

Table 1 Physical effects and properties of the link

These mainly physical parameters will be fixed in both schemes, whereas the bit rate and other system dependent parameters will be varied. The modulation scheme that gives a longer propagation distance will then be chosen as the better one of the two.

4.2 Origins of errors in Pulse position modulation transmission: comparing their importance in various schemes

As mentioned before, Gordon-Haus jitter is the most important phenomenon in a soliton link, in the absence of pulse interaction. It is necessary to compare the effect of GHJ and pulse interaction in our model links. There is one detail that should be mentioned in PPM link which is not exist in an OOK link. That is the additional degree of freedom in the form of the modulation index m. Unless specified otherwise, I will always choose the value of m near the value that can make the link achieve the maximum propagation distance (see below).

Figure 14 shows in the situation that soliton duration T_S= 4 ps, the main origins of error in PPM transmission (the corresponding curves for the case of OOK transmission are shown in [4]): the propagation distance L dependences of the GHJ width σ and the pulse shift due to soliton interaction τ_2.

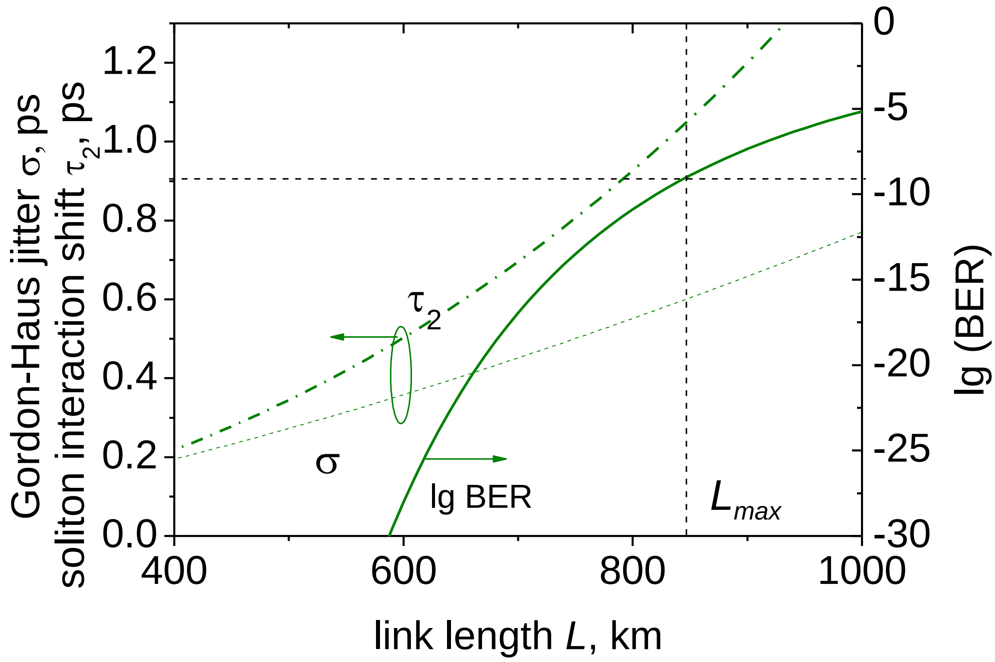

Figure 14 origins of error in PPM transmission. The modulation index is m= 0.75

It is seen that at propagation distances above a few tens of km, the two characteristic times are of the same order; in fact $\tau_2 > \sigma$, so that both soliton interaction and GHJ are important in determining the error probability (also shown in *Figure 14*). The point at which this parameter is big enough to satisfy (26) is the maximum transmission distance for the given parameters. The same logic is applied to OOK, using (19).

4.3 Solitonic link properties as function of the bit rate

In this section, I will focus on the solitonic link length dependence on transmission bit rate. Firstly, the dependence of pulse position modulation propagation distance on modulation index for different transmission bit rate will be introduced. Then, I will show the comparison of on-off keyed link and pulse position modulation link for different bit rate; furthermore, the predictions for the solitonic link with or without soliton interaction for both schemes will be compared.

4.3.1 Propagation distance of pulse position modulation affected by modulation index for different bit rate

The modulation index is a significant parameter for a pulse position modulation (PPM) transmission link. In high bit rate solitonic link, the affection of different modulation index will make difference while the transmission bit rate is changing.

With a low modulation index *m* (a wide guard band), the separation between the closest pulses in the PPM scheme is relatively broad. So the PPM suffers less from soliton interaction. However, because the pulse slots in the PPM scheme are unavoidably smaller, the small *m* values mean that it is easier for the pulse to stray into the wrong slot because of GHJ.

As the modulation index becomes large, on the other hand, the pulse slots become broader and the link length possibly increases up to a limit, in this case the PPM scheme may have advantages. However, for large values of *m*, approaching 1, the guard band becomes too small to prevent soliton interaction, so that the maximum propagation distance becomes to decrease before this advantage can be realised.

Therefore, it can be expected that there is an optimum value of *m* for each propagation bit rate, which is the value mentioned in the introduction to this chapter.

In order to study this effect, we calculate the propagation distance for different modulation index *m* and different transmission bit rates, mainly focusing on *m*= 0.5 ~ 0.8. In order to make direct comparison, *Figure 15* was plotted to show the effect of modulation index.

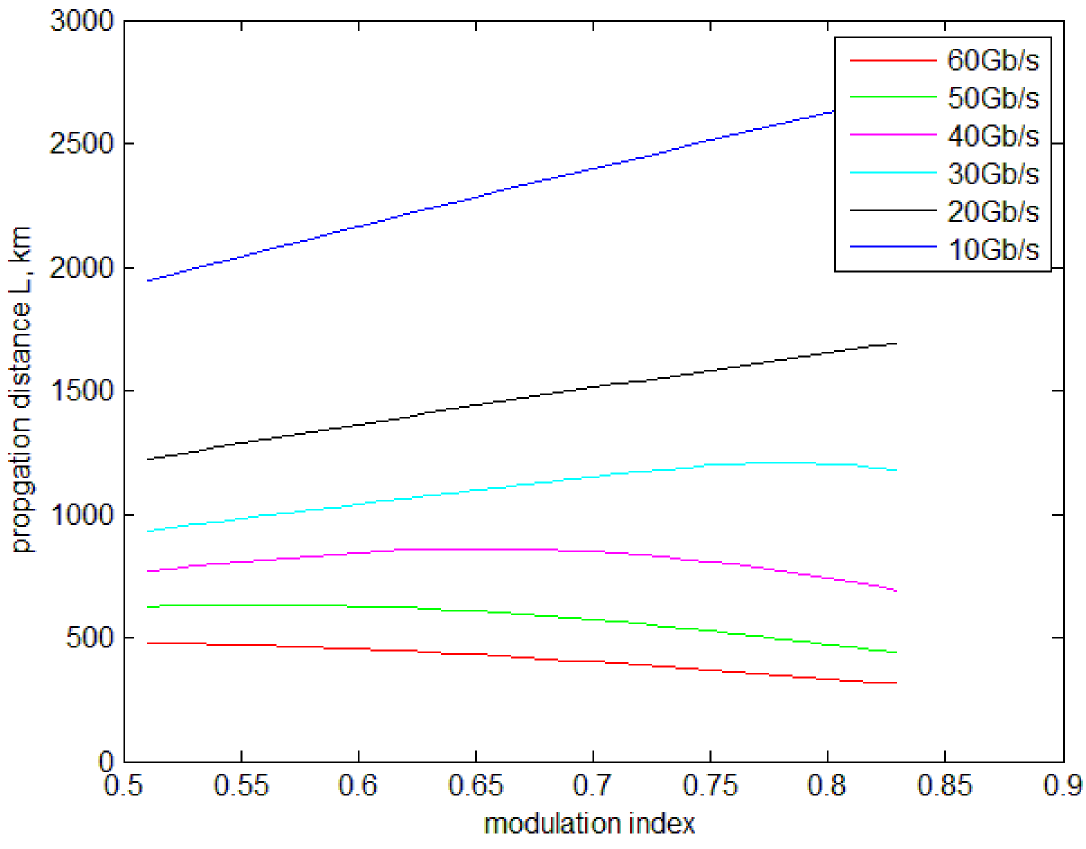

Figure 15 Propagation distance depend on modulation index for high bit rates in a PPM solitonic link

Figure 15 shows that with the transmission bit rate increases, the propagation distance decreases quite fast. This is because when the modulation index for each bit rate is identical, the pulses transmitted in a lower bit rate occupy broader slots than those in high bit rate. Meanwhile, the entire frame is also larger. In this case, the pulses are further apart than those transmitted at a higher bit rate. As a result, soliton interaction is also smaller. As mentioned before, that other parameters are the same, which means all amplifiers in those links are in common. Therefore, GHJ in all solitonic links has the same rms magnitude σ, so the probability of error at low bit rates is lower.

Soliton interaction reduces the propagation distance when pulses become closely enough spaced. As seen in *Figure 15*, this happens in two cases – either when the bit rate is high and/or when the modulation index is high.

Then I can summarise that the possible link length goes shorter as the transmission bit rate increases. This is not only for pulse position modulation system, but also it is common for all modulation schemes.

The next conclusion from *Figure 15*, is that there is a maximum in the dependence of the propagation distance on the modulation index. For B= 30Gb/s and 40Gb/s, this maximum is within the range of m sown in the figure, which agrees with the qualitative explanation given above. However, this does not mean there is not a maximum for other situation; it may achieve the maximum for a smaller modulation index for 50Gb/s and 60Gb/s; and for 20Gb/s, the modulation index could be approaching 1 to achieve this maximum. But for 10Gb/s, the separation between pulses are too large which makes negligible soliton interaction; so the link length is only affect by GHJ. Consequently, the curve is a monotonically growing line.

4.3.2 Comparison of on-off keying and pulse position modulation for different transmission bit rate

In this section, I will compare the maximum propagation distance of OOK and PPM scheme for different transmission bit rate B values. This will not only focus on soliton interaction case, but also non-interacting soliton case will be presented. In this way, not just the relative advantages of both schemes can be compared, but also the effect of soliton interaction on the solitonic link will be studied.

Based on *Figure 15* showed above, we can start with 40 Gb/s as this bit rate showed the features of the curve investigated (such as the optimum m) most clearly. Then we can compare this with the performance of systems with B values down to 25Gb/s and 10Gb/s, which show somewhat different behaviour.

Here, I choose soliton duration T_s= 4ps.

Performance at high bit rate cases (above 40Gb/s) is qualitatively similar to that of the 40Gb/s case; however it has to be noted that in the case of very high bit rates, the slot duration in the PPM scheme becomes comparable to the solution duration used in the simulations, so the calculations may become less accurate.

4.3.2.1 Comparison of the 40Gb/s case for both modulation schemes.

As mentioned before, here I will show the results of 40Gb/s for both modulation schemes, the maximum transmission distance are shown in the figure below. Also the effect of soliton interaction can be studied.

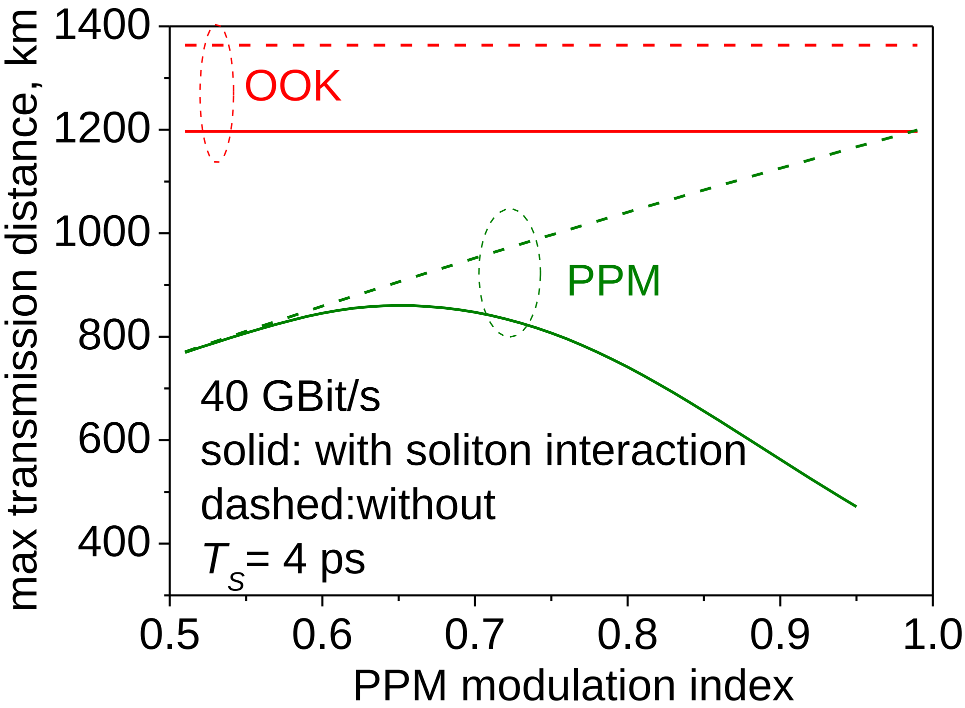

Figure 16 Maximum propagation distance for the PPM and OOK schemes with and without soliton interaction taken into account. (B= 40Gb/s; T_s= 4ps.)

Figure 16 illustrates the effect of modulation index on the maximum transmission distance in a pulse position modulation system. In agreement with the qualitative considerations presented above, there is an optimum value (in this case m≈0.65) which balances the effect of the pulse interaction and GHJ. At B= 40 GBit/s (and above), the effects are of the same order (as shown in Figure 14), so the maximum is well pronounced. Unfortunately, it prevents the PPM scheme from achieving any advantage over OOK modulation. Indeed, with a wide guard band (low modulation index), the pulses in the PPM scheme are more broadly spaced than in OOK, and so the PPM suffers less from soliton interaction than OOK. However, no advantage is achieved over OOK because the pulse slots in the PPM scheme are unavoidably smaller than in the case of OOK, particularly for small m values. As the modulation index increases, the pulse slots become broader and the link length possible increases up to a limit, giving some hope for an advantage over OOK. However, for large values of *m*, approaching 1, the guard band becomes too small to prevent soliton interaction, so that the maximum propagation distance becomes to decrease before this advantage can be realised. The L_{max} peaks at m~ 0.7 with the maximum propagation distance approximately 900 km. At the same time, the solitonic link for OOK shows a stable transmission distance around 1200 km which is shown schematically as a horizontal line in *Figure 16* (since *m* does not apply to OOK modulation).

The next step is to research is effect of soliton interaction. We can clearly see the sensitivity of pulse position modulation scheme. With the modulation index increases, the propagation distance decreases very fast. However, no matter how soliton interaction affect to the link, even there is no interaction; PPM scheme shows no advantage over OOK (shown as dashed curves).

4.3.2.2Comparison of the 25Gb/s case for both modulation schemes.

I next plot a figure for a lower transmission bit rate of 25Gb/s to see the difference from high bit rate (40Gb/s). Then the propagation distance of both modulation schemes are shown as:

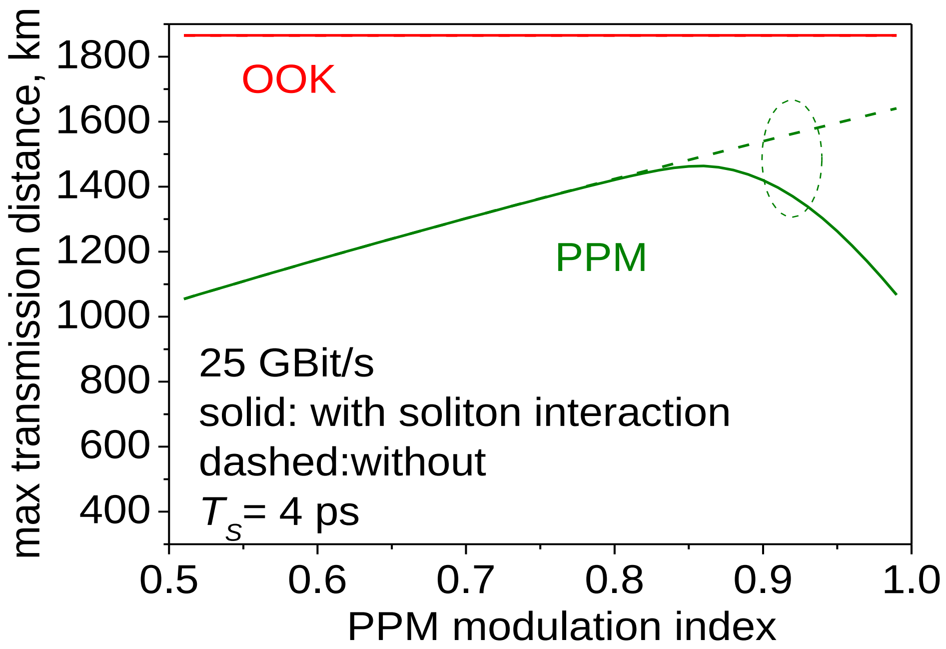

Figure 17 Maximum propagation distance for the PPM and OOK schemes with and without soliton interaction taken into account. (B= 25Gb/s; T_s= 4ps.)

Qualitatively, the behaviour is the same as in *Figure 17*, but the transmission distance has become longer for both PPM and OOK case than in the case of a 40 GBit/s system, and the maximum modulation index *m* for PPM has shifted to higher values. From *Figure 17* we can see that PPM scheme can get the maximum propagation distance around 1500km at *m*~ 0.85. Meanwhile, OOK scheme gets to nearly 1900km. It is still seen that PPM scheme has no advantage over OOK scheme.

Furthermore, PPM scheme still shows the effect of soliton interaction; even it is relatively smaller than in the 40Gb/s case. But here we see that there is no difference of OOK scheme either with or without soliton interaction. This is because when *B* decreases, the pulses in the solitonic link become much more broadly spaced than they are in high bit rate; as a result, even the solitons still interact each other, the effect are not as strong as when they are transmitted at a higher bit rate.

4.3.2.3 Comparison of the 10Gb/s case for both modulation schemes.

From 25Gb/s case I mentioned that the effect of soliton interaction is still of considerable importance, which however is reduced compared to the case of a higher transmission bit rate. In order to study this effect further, it is instructive to research transmitting information in an even lower bit rate – 10Gb/s. We then find out the transmission distance dependence on *m* as shown below:

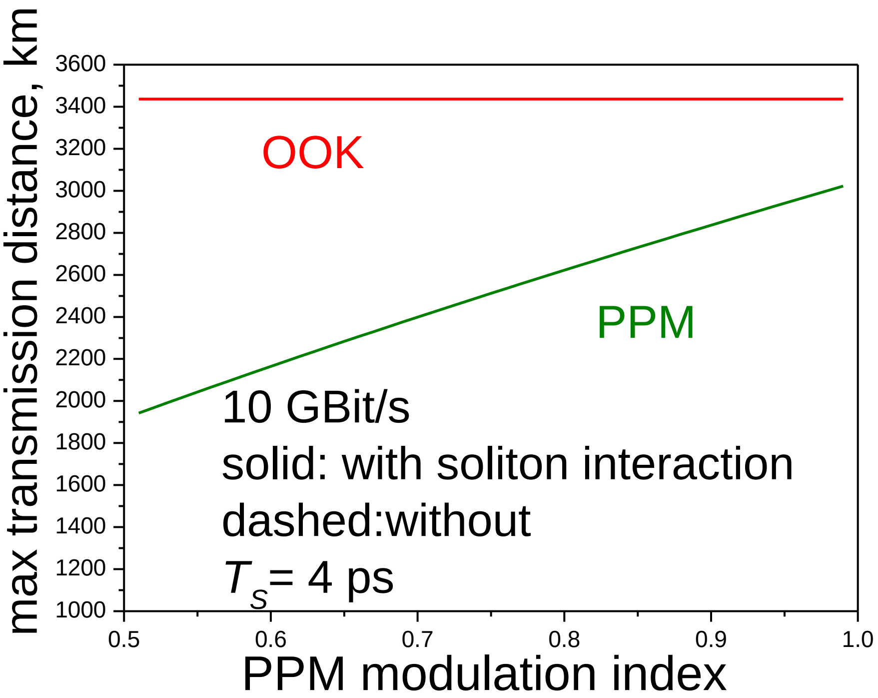

Figure 18 Maximum propagation distance for the PPM and OOK schemes with and without soliton interaction taken into account. (B= 10Gb/s; T_s= 4ps.)

Figure 18 still shows that the OOK provides a higher transmission rate than PPM at all values of *m*. Here, with the bit rate 10Gb/s, the maximum propagation distance for OOK is over 3400km when PPM is still below 3000km.

There are two differences from the cases of 10 and 40 GBit/s. Firstly, in the case of 10 GBit/s, the pulses are so broadly spaced that soliton interaction shows no appreciable effect on the transmission distance at any value of m, so the dashed curves showing the propagation distance without soliton interaction more or less coincide with the solid ones, which show the value for the case of the interaction present. Secondly, there is no observable maximum in the dependence $L_{max}(m)$, or in other words, optimum modulation index becomes equal to one – in the case of a low bit rate, there is no need in a guard band, just one slot duration is enough to prevent soliton interaction decreasing the propagation length.

This limiting case was investigated previously in [10].

4.3.2.4Comparison of the 80Gb/s case for both modulation schemes.

In order to make sure the conclusions are correct, we decide to compare this in an even high bit rate case of 80Gb/s, which is widely predicted to be the next bit rate in optical communications. Here, I will analyse several values of the PPM order and choose soliton duration shorter than that typically analysed for shorter bit rates because we need to ensure that a soliton does not penetrate the neighbouring slots. The maximum propagation distance and optimum modulation index m are shown in the table below.

PPM order, M	Maximum transmission distance, $L_{max(PPM)}$, km	Optimum modulation index, m	Maximum transmission distance, $L_{max(OOK)}$, km
1	444	0.9	
2	406	0.8	541
3	350	0.84	

Table 2 Maximum transmission distance for 80Gb/s case, T_s = 1ps.

Table 2 show the same qualitative tendency as those presented earlier for lower bit rates: compared to OOK, PPM has still no advantage in this case.

Next, we can increase the soliton duration to 2 ps. The results are shown in *Table 3*.

PPM order, M	Maximum transmission distance, $L_{max(PPM)}$, km	Optimum modulation index, m	Maximum transmission distance, $L_{max(OOK)}$, km
1	449	0.66	
2	368	0.54	426
3	373	0.66	

Table 3 Maximum transmission distance for 80Gb/s case, T_s = 2ps.

The table above shows that the maximum propagation distance of PPM scheme depending on the PPM order M does not decrease monotonically. This is because the maximum transmission distance at M=2 is achieved at a lower modulation index.

Table 3 also shows that during PPM order is 1, soliton duration is 2ps; the PPM scheme can achieve a higher transmission distance than OOK. Thus is some situation PPM scheme can be better than OOK scheme, though only marginally and only for a narrow range of pulse durations.

4.3.2.5Maximum transmission distance dependence on bit rate for on-off keying and pulse position modulation

In this section, I will give the exact comparison of the maximum transmission distance depending on bit rate when soliton duration T_S = 4 ps. For PPM scheme, I will always choose the certain modulation index which leads the propagation distance to be the maximum.

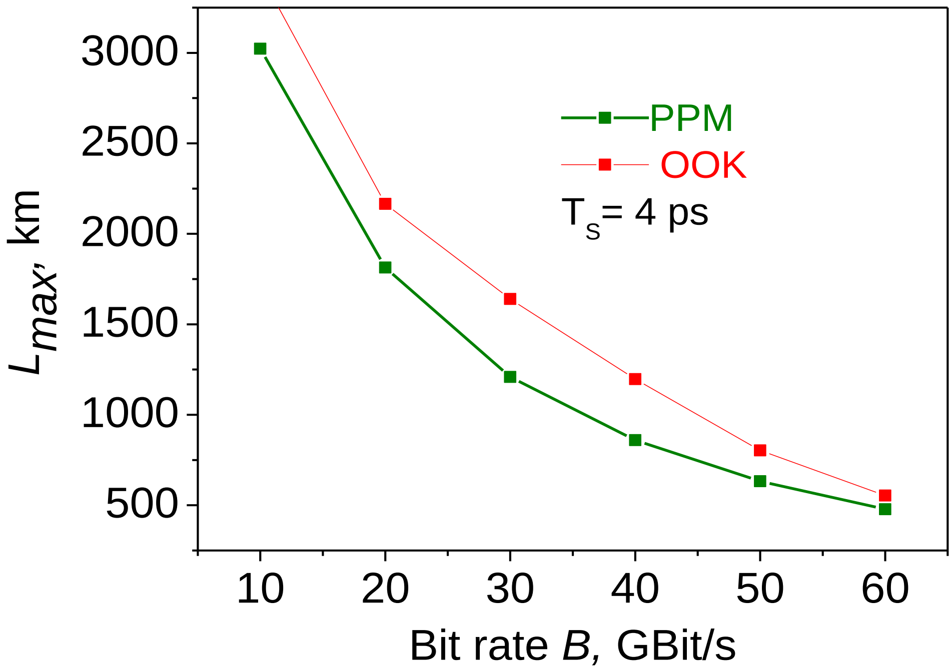

Figure 19 Maximum propagation distance for on-off keying and pulse position modulation depend on the transmission bit rate. (T_S = 4 ps)

As shown in *Figure 19*, one can conclude that under all conditions covered by the simulations presented in it, there is no advantage for pulse position modulation scheme over on- off keying scheme as the transmission bit rate is varied

4.3.3 Comparison of on-off keying and pulse position modulation for different soliton duration values

In this section, I will compare the dependence of soliton duration on the maximum propagation distance for on-off keyed link and pulse position modulation link. Soliton duration is a quite significant phenomenon in solitonic link. In fact, the propagation distance is determined by the soliton duration.

We investigate soliton duration varying from about 1 ~ 5 ps with the step every 0.5 ps. And the transmission bit rate is 40Gb/s which can show the sensitivity of pulse position modulation scheme. Then the distribution of maximum transmission distance depends on soliton duration is shown as:

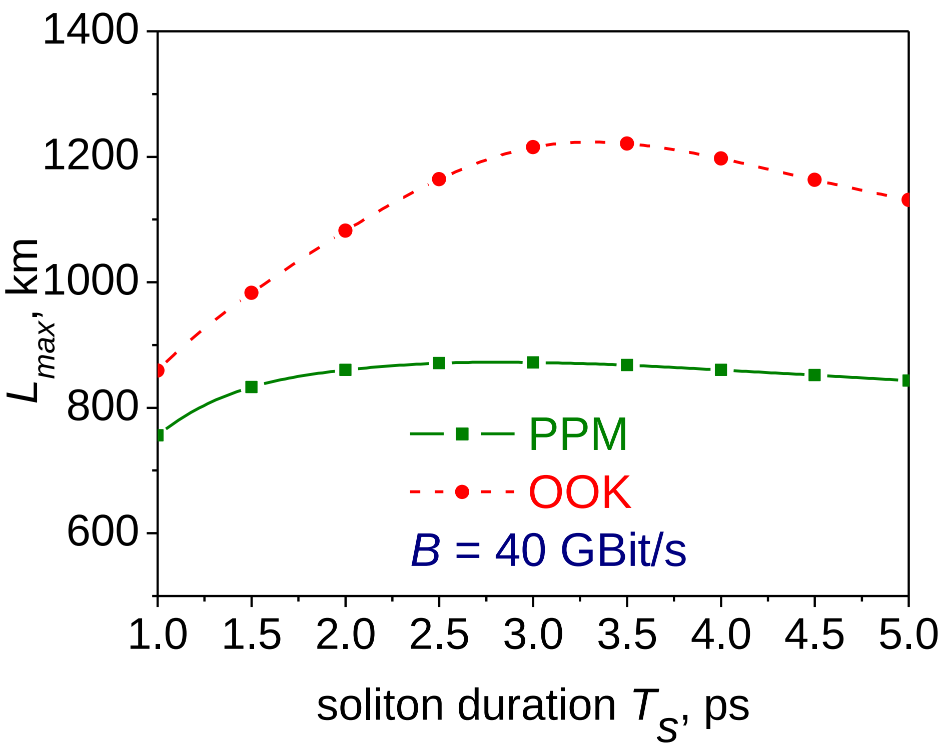

Figure 20 Maximum transmission distance depend on soliton duration (B= 40Gb/s)

Figure 20 shows the dependence of soliton duration on solitonic link represented by maximum propagation distance. And at each point PPM scheme shows no advantage over OOK scheme regardless of the soliton duration. It also implies that the PPM is not a good scheme for transmitting information at high bit rates. Indeed, high bit rates mean a small optimum value of the modulation index m, which means small slot durations, which means soliton pulses should be short not to overlap with the neighbouring slots. As seen in the figure above, all this leads to short transmission distance, making the PPM compare even less favourably with OOK than at moderate bit rates.

4.3.4 Effect of the order of the PPM scheme on the maximum propagation distance: comparison of the cases of *M*=1, *M*=2, and *M*=3.

As mentioned in section 3.2, all the calculations so far have been performed for the second-order PPM (M=2), since this was the optimum for the case without soliton interaction [10]. We need to confirm that this remains the case with the soliton interaction included. I will compare the effect of the order of pulse position modulation by changing it from 1 to 3. Then, the consideration of which is better for high transmission bit rate will be based on the maximum transmission distance, as above.

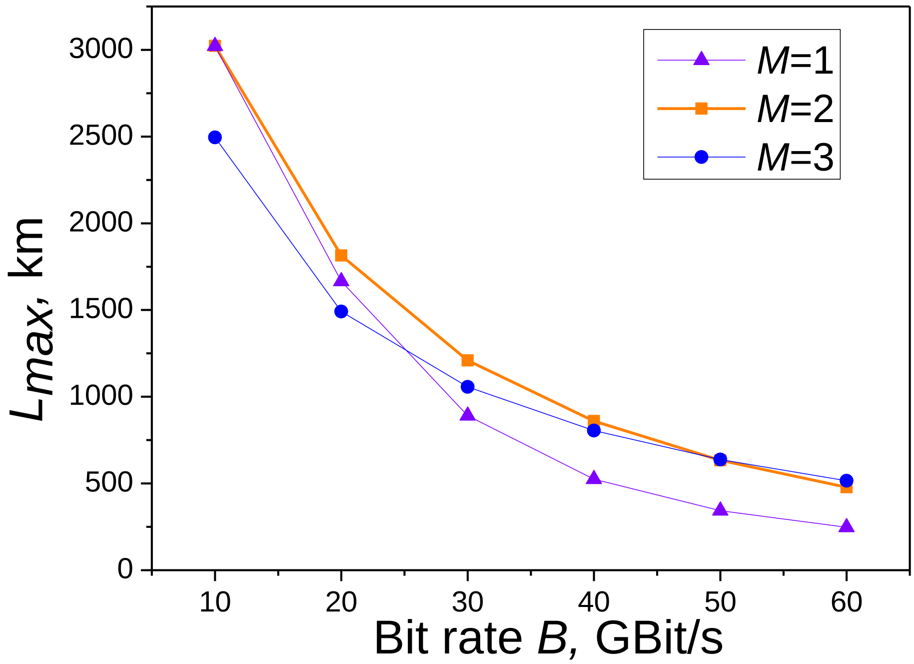

Figure 21 Maximum propagation distance depend on PPM order M=1, 2, 3 (T_s= 4 ps)

Figure 21 shows the dependence of maximum transmission distance on the order of pulse position modulation (M= 1, 2, 3). In all cases, the modulation index m is chosen as the optimum value to maximize L_{max}.

The results may be seen as somewhat counterintuitive, because in the case M= 1 there are only two slots in each frame; which gives a possibility that when the PPM order is smaller, the soliton interaction could be relatively small. Correspondently, one could expect the transmission distance to be longer than in the case of higher values of M. However, the figure above shows that there is no advantage in using the order M= 1, presumably because in this case the pulse frame itself has to be shorter than in the case of M= 2.

The results calculated for M= 3 show a shorter propagation distance than that calculated for M= 2 at low to moderate (up to about 50 Gb/s) bit rate values; as the bit rate is increased to 50Gb/s, the propagation distance becomes longer than in the case of M= 2 but only just. This justifies the choice of M=2 in all the calculations presented in previous chapters and sections.

Chapter 5 PPM and OOK in a capacity constrained non-solitonic RZ link.

I will briefly consider the applicability of PPM as opposed to OOK in *non-solitonic* return to zero capacity constrained links in this chapter.

All the earlier studies on the high bit rate PPM concentrated on solitonic links, which do indeed have their advantages, chiefly allowing much longer propagation distances than linear RZ short pulses, because of suppressed dispersion.

However, a number of recent developments made non-solitonic methods of countering the effects of dispersion a very attractive alternative. Most importantly, group velocity dispersion compensation in optical communications has seen rapid development in recent years, making linear, non-solitonic RZ links feasible. Since there is no nonlinearity involved in such a link, there is no interrelation between pulse amplitude, duration, and instantaneous frequency which characterises a soliton [17], so there is no Gordon-Haus Jitter. As a result, compared to a solitonic link, a non-solitonic link can in principle achieve longer propagation distance if the group velocity dispersion compensation is ideal or nearly ideal. In a linear regime, the initial waveform is completely recovered by a GVD compensator, eliminating effects of both distortion and noise potentially compensating up to 100% of the dispersion effects [18]. The linear, non-solitonic optical fibre link also has the advantage of being better compatible with Wavelength Division Multiplexing (WDM) technology, because in a WDM link is important to have no interaction between channels at different wavelengths.

Since there is no soliton here, we can approximate the pulse shape by a Gaussian and use the formula for the duration of a Gaussian pulse in a dispersive medium. This is known to increase, in the case of uniform dispersion, with propagation distance; the formula describing this growth can be found, for example, in [19] in the form of

$$\tau_p = \sqrt{\tau_0^2 + \left(\frac{\beta_2 z}{\tau_0}\right)^2} \tag{27}$$

, where τ_0 is the initial pulse duration; β_2 is the group-velocity dispersion parameter (GVD). In the case of a link with dispersion compensation, either through a non-constant β_2 (e.g. by interspersing fibre stretches with positive and negative dispersion) or by using a lumped dispersion compensating element, it can be generalised as:

$$\tau_p = \sqrt{\tau_0^2 + \left(\frac{\int \beta_2 dz - D_{compensator}}{\tau_0}\right)^2} \tag{28}$$

, where $D_{compensator,}$ is the total dispersion of a *lumped* dispersion compensator, in ps^2; and the limits of integration are over the entire link length.

So long as there is no nonlinearity in the link, the spectral width of a wave packet remains constant in this linear medium. From (28), we can get an approximation for the transmission distance if we require that the pulse does not get broadened enough to cause intersymbol interference, or fill a significant part of a neighbouring slot. For definiteness and simplicity, we can set a condition for that as:

$$\tau_p = \sqrt{2} T_{slot} \tag{29}$$

Then, from (28), the maximum transmission length can be achieved when the value of the initial pulse duration is the same as the slot interval. In this simplest case, the maximum propagation distance is estimated as:

$$L_{\max} = \frac{T_{slot}^2 + D_{compensator}}{|\beta_2|} = L_D\big|_{\tau = T_{slot}} + \frac{D_{compensator}}{|\beta_2|} \tag{30}$$

, where as before, L_D is the dispersion length.

Assuming the expression (30) is used for the same transmission medium for PPM and

OOK, L_D is given by the same expression for either OOK or PPM (but with a different T_{slot}). The value of $D_{compensator}$, is also identical. β_2 is GVP which is fixed for the link of both scheme. So, the maximum transmission distance L_{max} depends on the pulse width $\tau_p = \sqrt{2}T_{slot}$. Then, one observes that the slot duration for pulse position modulation is $T_{slot,\,PPM} = {(1-m)T_{frame}}/{2^M}$; and which for on-off keying is $T_{slot,\,OOK} = 1/B$. Clearly, $T_{slot,\,PPM} < T_{slot,\,OOK}$. Hence, $L_{\max}^{(PPM)} < L_{\max}^{(OOK)}$, and so PPM is not competitive in this case either.

Hence, we have to conclude that PPM is not competitive in any capacity constrained link. The probable way which could give PPM an advantage than OOK needs either no dispersion or fully compensated dispersion, in which case the link is sensitivity rather than capacity constrained.

Chapter 6 Some considerations about possible implementations of a high bit rate PPM.

In this chapter, I will briefly consider the possible implementation of pulse position modulation in a high bit rate link, whether sensitivity or capacity constrained. This will be compared with the current implementation of an OOK link.

To realize PPM at high bit rates, a designer can either try to delay an individual pulse in each frame, or use several pulse streams, already delayed by the right amount, and select a pulse from one of them in each frame.

It is technologically difficult to implement widely tunable (by more or less the entire frame widths) delays by tens of pica-seconds, so I think the latter case, selecting between delayed pulses, is more promising. A possible implementation may look as follows:

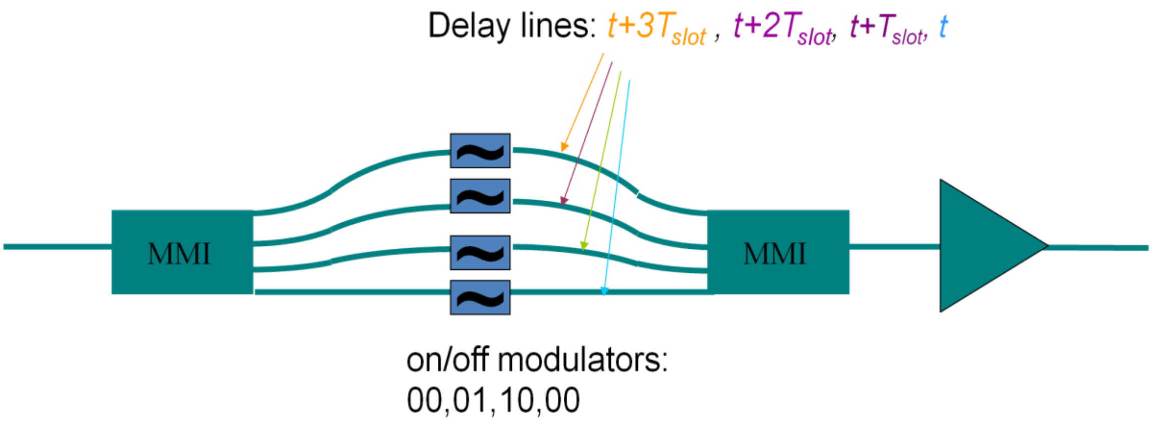

Figure 22 Possible multi-GHz integrated PPM implementation

Figure 22 shows that there are four modulators and an amplifier in between which ensure the system can be realistically implemented. The modulators are used for picking the pulses; one pulse in a certain frame can only go through one of the four modulators randomly. The MMI multiplexes and demultiplexes messages. The delay lines give pulse streams with the required delay; and the modulators are used for choosing one of those delay lines, it requires that each pulse frame can only go through one modulator and also must have one. At the end of *Figure 22* there is an amplifier compensates used for the splitting loss.

For comparison, a current implementation of OOK at high bit rates is shown in *Figure 23*.

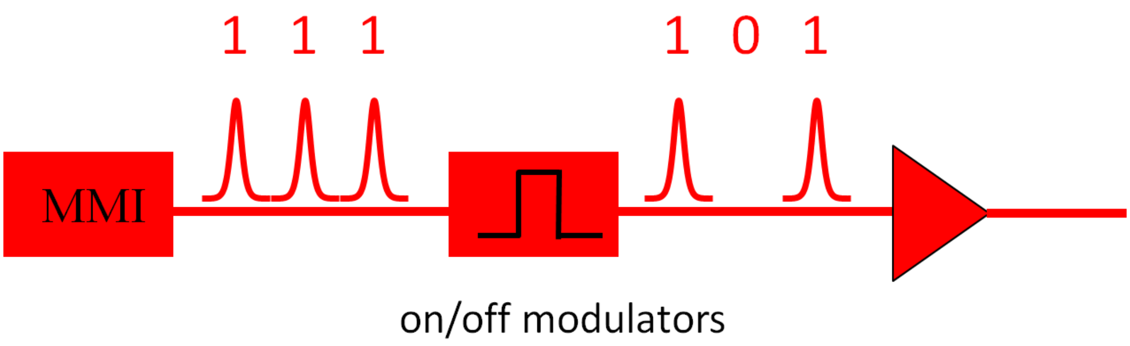

Figure 23 Multi-GHz integrated OOK implementation

Figure 23 shows a generic on-off keying implementation in high bit rate, which has a key in between to decide the output is '1' or '0'. The implementation is easier than that of PPM, but uses fundamentally the same technology.

Chapter 7 Conclusions and Future Work

In this chapter, the conclusions of the whole project will be firstly introduced. And then discussion of possible future work will be given.

7.1 Conclusions

I have developed a simple, mainly analytical expression for the effect of soliton interaction on the maximum link distance for solitonic links using either on-off keying or pulse position modulation. The soliton interaction substantially decreases the maximum transmission distance achievable in both schemes and also makes for a non-monotonic dependence of the transmission distance on the modulation index m in the case of PPM, with the longest transmission distance typically achieved at $m = 0.6 \sim 0.8$ depending on the bit rate and soliton duration. For most conditions studied, the maximum transmission distance for the case of OOK modulation remained longer than for PPM. At very high bit rates, for certain soliton pulse durations, there was a narrow range of parameters (bit rates and soliton pulse durations) in which solitonic PPM performance was (marginally) better than that of OOK.

7.2 Future Work

This project was set to verify whether pulse position modulation scheme could achieve a higher propagation distance than an on-off keying scheme. Then the combination of pulse position modulation with wavelength division multiplexing (WDM) could be discussed depending on this expectation. Unfortunately, the result was in nearly all cases negative: at high bit rates, which tend to be capacity constrained, the PPM scheme was less promising than OOK. However, in a narrow range of soliton duration at very high bit rates (80 GBit/s), PPM can compete with OOK.

One aspect of the PPM link we have not covered in the research was the performance of the receiver scheme, including both electronics of the photoreceiver and the response of the photodetector used in the receiver. In all our calculations, as in the previous theoretical analysis [9], [10], it was assumed that the received is infinitely fast and no pulse distortion takes place. The next stage of this work could revisit this analysis with a more realistic model of the receiver and see whether PPM (where pulses are spaced further apart than in the case of OOK) can reduce the requirements on the receiver optics and electronics.

Appendix The soliton units

The mathematical description of solitons employs the nonlinear Schrödinger (NLS) equation, which satisfied by the pulse envelope $A(z, t)$ in the presence of GVD and SPM. And this is a dimensional explanation of equation (5). Following the notations of the textbook[1] which in turn follows the original publications[2], this equation can be written as

$$\frac{\partial A}{\partial z}+\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2}-\frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3}=i\gamma\left|A\right|^2 A-\frac{\alpha}{2}A$$

, where the α parameter includes the fibre loss in cm^{-1} or m^{-1}, and β_2 and β_3 account for the second- and third-order dispersion effects (essentially, the frequency dependence of the dielectric constant of the fibre core). The nonlinear parameter $\gamma = 2\pi\, n_2/ (\lambda A_{eff})$ is defined in terms of the nonlinear-index coefficient n_2, the optical wavelength λ, and the effective area of the waveguide mode cross-section A_{eff}.

The exact soliton solutions are obtained when $\alpha = 0$ and $\beta_3 = 0$. Then, the NLS equation is re-written in a normalized form by introducing

$$\tau=\frac{t}{T_0},\quad \xi=\frac{z}{L_D},\quad u=\frac{A}{\sqrt{P_0}}$$

, where T_0 is a measure of the pulse width, P_0 is the peak power of the pulse, and $L_D = T_0^2 /|\beta_2|$ is the dispersion length. These are the units used in Equations (5)-(8).

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List of Acronyms

BER	Bit Error Rate
DPSK	Differential Phase-Shift Keying
GHJ	Gordon-Haus jitter
GVD	group velocity dispersion
OOK	On- Off Keying
PPM	Pulse Position Modulation
pdf	probability density function
RF	radio frequency
SNR	Signal Noise Ratio
SPM	self-phase modulation
WDM	Wavelength Division Multiplexing

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